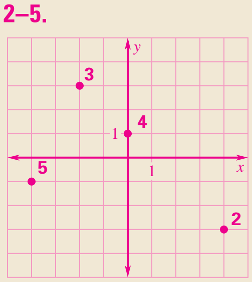
PLAN AND PREPARE

Main Ideas

In this chapter students use properties of midsegments to find lengths of segments in triangles. They then learn to write a coordinate proof. They explore perpendicular bisectors and use the concurrency of perpendicular bisectors of a triangle to solve problems. They use angle bisectors to find distance relationships and explore the concurrency of angle bisectors of a triangle. Students use medians of a triangle to find the centroid and to find segment lengths, and they use altitudes of a triangle to find and explore the orthocenter. Students relate side length and angle measures of a triangle, find possible side lengths for the third side of a triangle, use inequalities to make comparisons in two triangles, and use the Hinge Theorem and its converse to solve multi-step problems. Finally, students learn to write indirect proofs.

2–5.



5 Relationships within Triangles

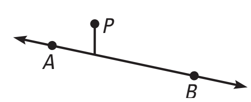
Before

Previously, you learned the following skills, which you'll use in this chapter: plotting points, finding distances, using properties of triangles, and solving inequalities.

Prerequisite Skills

VOCABULARY CHECK

1. Is the *distance from point P to line AB* equal to the length of $\overline{PQ}$? *Explain* why or why not.
   No; PQ is not perpendicular to AB.



SKILLS AND ALGEBRA CHECK

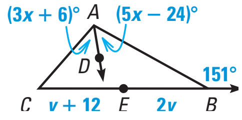
Plot the point in a coordinate plane. **2–5. See margin.**

2. $(4, -3)$ 3. $(-2, 3)$ 4. $(0, 1)$ 5. $(-4, -1)$

$\triangle PQR$ has the given vertices. Find the length of each side. Then classify the triangle by its sides.

6. $P(2, 0)$, $Q(6, 6)$, and $R(12, 2)$ **$PQ = \sqrt{52} \approx 7.2$, $QR = \sqrt{52} \approx 7.2$, $PR = \sqrt{104} \approx 10.2$; isosceles**
7. $P(2, 3)$, $Q(4, 7)$, and $R(11, 3)$ **$PQ = \sqrt{20} \approx 4.5$, $QR = \sqrt{65} \approx 8.1$, $PR = 9$; scalene**

In the diagram, $CE = EB$ and $m\angle CAD = m\angle BAD$. Find the specified measurement.



8. CE **24** 9. $m\angle BAC$ **102°**

Solve.

10. $43 > x + 35$ **$x < 8$** 11. $-14 < x + 9$ **$x > -23$** 12. $x + 26 \le 54$ **$x \le 28$**

Chapter Planning Guide

Chapter Resource Book
- Teaching Guide/Lesson Plan
- Project with Rubric

Assessment and Intervention
- Assessment Book
- Benchmark Tests
- Remediation Book
- Skills Readiness

Interactive Technology
- Power Presentations
- Activity Generator
- Animated Geometry
- ExamView™ Assessment Suite
- Online Quizzes
- eEdition
- @HomeTutor

Resources for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

Now

In this chapter, you will apply the big ideas listed below and reviewed in the Chapter Summary. You will also use the key vocabulary listed below.

Big Ideas

1. **Using properties of special segments in triangles**
2. **Using triangle inequalities to determine what triangles are possible**
3. **Extending methods for justifying and proving relationships**

KEY VOCABULARY

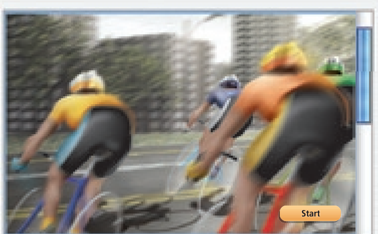
- midsegment of a triangle
- coordinate proof
- perpendicular bisector
- equidistant
- point of concurrency
- circumcenter
- incenter
- median of a triangle
- centroid
- altitude of a triangle
- orthocenter
- indirect proof

Why?

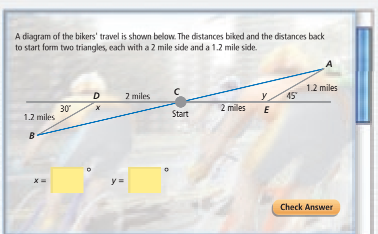
You can use triangle relationships to find and compare angle measures and distances. For example, if two sides of a triangle represent travel along two roads, then the third side represents the distance back to the starting point.

Animated Geometry

The animation illustrated below helps you answer a question from this chapter: After taking different routes, which group of bikers is farther from the camp?



Two groups of bikers head out from the same point and use different routes.



Enter values for x and y. Predict which bikers are farther from the start.

Animated Geometry at my.hrw.com

Differentiated Instruction Resources

- Reading Strategies
- Differentiated Instruction Lesson Notes
- English Learners Lesson Notes
- Inclusion Lesson Notes
- Teaching Strategies with Sample Worksheets
- Using Technology in the Classroom
- Tips for New Teachers
- Math Background Notes
- Assessment Strategies
- Teacher Survival Activities
- Bulletin Board Idea

Prerequisite Skills

Skills Readiness, available online, provides review and practice for the Skills and Algebra Check portion of the Prerequisite Skills quiz.

How student answers the exercises	What to assign from *Skills Readiness*
Any of Exs. 2–5 answered incorrectly	**Skill 79** Graph ordered pairs
Any of Exs. 6–7 answered incorrectly	**Skill 73** Classify triangles
Any of Exs. 8–9 answered incorrectly	**Skill 69** Solve multi-step equations
Any of Exs. 10–12 answered incorrectly	**Skill 74** Solve inequalities
All exercises answered correctly	Chapter Enrichment

Additional skills review and practice is available in the Skills Review Handbook and the @HomeTutor.

Investigate Segments in Triangles

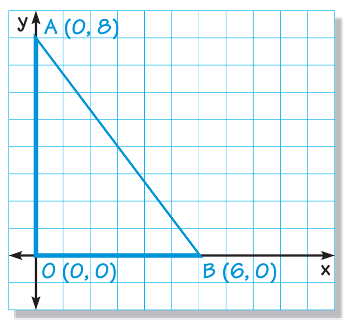
MATERIALS • graph paper • ruler • pencil

QUESTION How are the midsegments of a triangle related to the sides of the triangle?

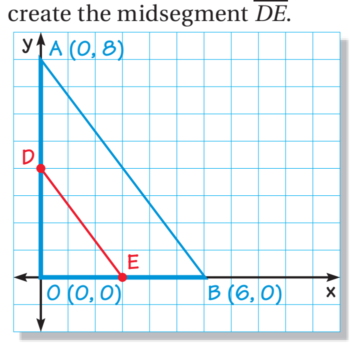
A *midsegment* of a triangle connects the midpoints of two sides of a triangle.

EXPLORE Draw and find a midsegment

STEP 1 ***Draw a right triangle*** Draw a right triangle with legs on the x-axis and the y-axis. Use vertices $A(0, 8)$, $B(6, 0)$, and $O(0, 0)$ as Case 1.



STEP 2 ***Draw the midsegment*** Find the midpoints of $\overline{OA}$ and $\overline{OB}$. Plot the midpoints and label them D and E. Connect them to create the midsegment $\overline{DE}$.



STEP 3 ***Make a table*** Draw the Case 2 triangle below. Copy and complete the table. **See margin.**

	Case 1	Case 2
O	(0, 0)	(0, 0)
A	(0, 8)	(0, 11)
B	(6, 0)	(5, 0)
D	?	?
E	?	?
Slope of $\overline{AB}$	?	?
Slope of $\overline{DE}$	?	?
Length of $\overline{AB}$	?	?
Length of $\overline{DE}$	?	?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Choose two other right triangles with legs on the axes. Add these triangles as Cases 3 and 4 to your table. **See margin.**
2. Expand your table in Step 3 for Case 5 with $A(0, n)$, $B(k, 0)$, and $O(0, 0)$. **See margin.**
3. Expand your table in Step 3 for Case 6 with $A(0, 2n)$, $B(2k, 0)$, and $O(0, 0)$. **See margin.**
4. What do you notice about the slopes of $\overline{AB}$ and $\overline{DE}$? What do you notice about the lengths of $\overline{AB}$ and $\overline{DE}$? **They are the same; $\overline{DE}$ is half the length of $\overline{AB}$.**
5. In each case, is the midsegment $\overline{DE}$ parallel to $\overline{AB}$? *Explain.* **Yes; they have the same slope.**
6. Are your observations true for the midsegment created by connecting the midpoints of $\overline{OA}$ and $\overline{AB}$? What about the midsegment connecting the midpoints of $\overline{AB}$ and $\overline{OB}$? **yes; yes**
7. Make a conjecture about the relationship between a midsegment and a side of the triangle. Test your conjecture using an acute triangle. **A midsegment of a triangle and the third side of the triangle are parallel and the midsegment is half the length of the third side.**

1 PLAN AND PREPARE

Explore the Concept

- Students will draw a midsegment of a triangle and compare it to the third side of the triangle.
- This activity leads into the study of midsegments in this lesson, Example 1.

Materials

Each student will need:

- graph paper
- ruler
- Activity Support Master (*Chapter Resource Book*)

Recommended Time

Work activity: 10 min

Discuss results: 5 min

Grouping

Students should work individually.

2 TEACH

Tips for Success

Suggest that students use the Midpoint Formula if they have difficulty finding the coordinates of D and E.

Key Questions

- How does $\overline{DE}$ compare to $\overline{AB}$? **It is half as long.**
- Is the midsegment relationship true in any triangle? **yes**

Alternative Strategy

Demonstrate how to do this activity on the overhead projector, record the results in a table, and ask students to make conjectures from the table.

Key Discovery

The segment connecting the midpoints of two sides of a triangle is parallel to the third side and half as long.

3 ASSESS AND RETEACH

Step 3, 1–3. See Additional Answers.

1. If one side of a triangle is 16 units long, how long is the midsegment of the other two sides? **8 units**

5.1 Midsegment Theorem and Coordinate Proof

Before You used coordinates to show properties of figures.

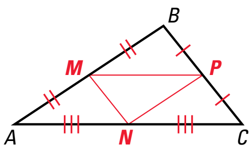
Now You will use properties of midsegments and write coordinate proofs.

Why? So you can use indirect measure to find a height, as in Ex. 35.

Key Vocabulary
- **midsegment of a triangle**
- **coordinate proof**

A **midsegment of a triangle** is a segment that connects the midpoints of two sides of the triangle. Every triangle has three midsegments.

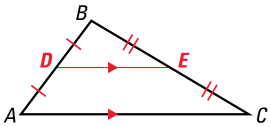
The midsegments of $\triangle ABC$ at the right are $\overline{MP}$, $\overline{MN}$, and $\overline{NP}$.



THEOREM *For Your Notebook*

THEOREM 5.1 Midsegment Theorem

The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.

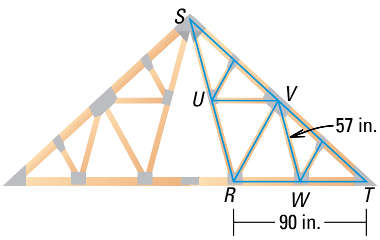


$\overline{DE} \parallel \overline{AC}$ and $DE = \frac{1}{2}AC$

EXAMPLE 1 **Use the Midsegment Theorem to find lengths**

READ DIAGRAMS
In the diagram for Example 1, midsegment $\overline{UV}$ can be called "the midsegment opposite $\overline{RT}$."

CONSTRUCTION Triangles are used for strength in roof trusses. In the diagram, $\overline{UV}$ and $\overline{VW}$ are midsegments of $\triangle RST$. Find UV and RS.



Solution

$$UV = \frac{1}{2} \cdot RT = \frac{1}{2}(90 \text{ in.}) = 45 \text{ in.}$$

$$RS = 2 \cdot VW = 2(57 \text{ in.}) = 114 \text{ in.}$$

GUIDED PRACTICE for Example 1

1. Copy the diagram in Example 1. Draw and name the third midsegment.
2. In Example 1, suppose the distance UW is 81 inches. Find VS. **81 in.**

1. $\overline{UW}$**; see Additional Answers.**

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

For Exercises 1–4, use $A(0, 10)$, $B(24, 0)$, and $C(0, 0)$.

1. Find AB. **26**

2. Find the midpoint of $\overline{CA}$. **(0, 5)**

3. Find the midpoint of $\overline{AB}$. **(12, 5)**

4. Find the slope of $\overline{AB}$. $-\frac{5}{12}$

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block
- See *Teaching Guide/Lesson Plan*.

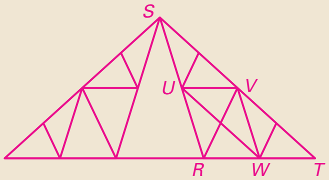
2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3

How do you write a coordinate proof? **Tell students they will learn how to answer this question by placing a figure in the coordinate plane, assigning coordinates to the vertices, and then using the midpoint, distance, and/or slope formulas.**

1.



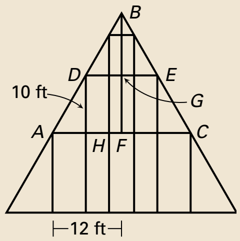
Motivating the Lesson

The sides of a playground swing set are inverted "A" frames, with a horizontal bar joining the supports at their midpoints. Tell students that this lesson investigates the segment that joins the midpoints of two sides of a triangle.

3 TEACH

Extra Example 1

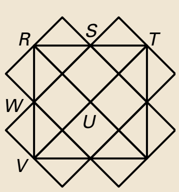
In the diagram of an A-frame house, $\overline{DG}$ and $\overline{DH}$ are midsegments of $\triangle ABF$. Find DG and BF.



$DG = 6$ ft; $BF = 20$ ft

Extra Example 2

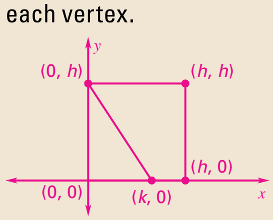
In the diagram, $\overline{RS} \cong \overline{TS}$ and $\overline{RW} \cong \overline{VW}$. Show that $\overline{VT} \parallel \overline{WS}$.



$\overline{RS} \cong \overline{TS}$ and $\overline{RW} \cong \overline{VW}$, so S and W are the midpoints of $\overline{RT}$ and $\overline{RV}$, and $\overline{SW}$ is a midsegment of $\triangle RTV$. $\overline{VT} \parallel \overline{WS}$ by the Midsegment Theorem.

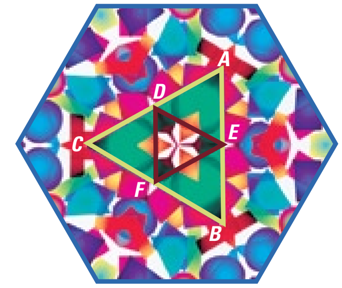
Extra Example 3

Place a square and a right triangle in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.



EXAMPLE 2 Use the Midsegment Theorem

In the kaleidoscope image, $\overline{AE} \cong \overline{BE}$ and $\overline{AD} \cong \overline{CD}$. Show that $\overline{CB} \parallel \overline{DE}$.



Solution

Because $\overline{AE} \cong \overline{BE}$ and $\overline{AD} \cong \overline{CD}$, E is the midpoint of $\overline{AB}$ and D is the midpoint of $\overline{AC}$ by definition. Then $\overline{DE}$ is a midsegment of $\triangle ABC$ by definition and $\overline{CB} \parallel \overline{DE}$ by the Midsegment Theorem.

COORDINATE PROOF A **coordinate proof** involves placing geometric figures in a coordinate plane. When you use variables to represent the coordinates of a figure in a coordinate proof, the results are true for all figures of that type.

EXAMPLE 3 Place a figure in a coordinate plane

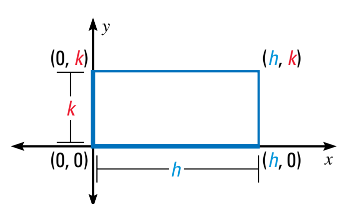
Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

a. A rectangle **b.** A scalene triangle

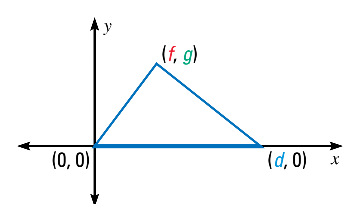
Solution

It is easy to find lengths of horizontal and vertical segments and distances from (0, 0), so place one vertex at the origin and one or more sides on an axis.

a. Let h represent the length and k represent the width.



b. Notice that you need to use three different variables.



USE VARIABLES
The rectangle shown represents a general rectangle because the choice of coordinates is based only on the definition of a rectangle. If you use this rectangle to prove a result, the result will be true for all rectangles.

Animated Geometry at my.hrw.com

✓ GUIDED PRACTICE for Examples 2 and 3

3. In Example 2, if F is the midpoint of $\overline{CB}$, what do you know about $\overline{DF}$?

4. Show another way to place the rectangle in part (a) of Example 3 that is convenient for finding side lengths. Assign new coordinates. **See margin.**

5. Is it possible to find any of the side lengths in part (b) of Example 3 without using the Distance Formula? *Explain.* **Yes; the length of one side is d.**

6. A square has vertices $(0, 0)$, $(m, 0)$, and $(0, m)$. Find the fourth vertex. **(m, m)**

3. $\overline{DF}$ is a midsegment of $\triangle ABC$, $\overline{DF} \parallel \overline{AB}$, and DF is half the length of AB.

Differentiated Instruction

Below Level Students who need more practice with midsegments should be instructed to draw examples of scalene, right, and obtuse triangles. Ask them to use a ruler to locate the midpoints, to draw the midsegments, and to verify that the midsegment is one half the third side. Then ask them to use a protractor to verify that the midsegment is parallel to the third side.

See also the *Differentiated Instruction Resources* for more strategies.

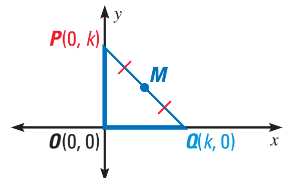
EXAMPLE 4 Apply variable coordinates

Place an isosceles right triangle in a coordinate plane. Then find the length of the hypotenuse and the coordinates of its midpoint M.

ANOTHER WAY
For an alternative method for solving the problem in Example 4, for the **Problem Solving Workshop**.

Solution

Place $\triangle PQO$ with the right angle at the origin. Let the length of the legs be k. Then the vertices are located at $P(0, k)$, $Q(k, 0)$, and $O(0, 0)$.



Use the Distance Formula to find PQ.

$$PQ = \sqrt{(k-0)^2 + (0-k)^2} = \sqrt{k^2 + (-k)^2} = \sqrt{k^2 + k^2} = \sqrt{2k^2} = k\sqrt{2}$$

Use the Midpoint Formula to find the midpoint M of the hypotenuse.

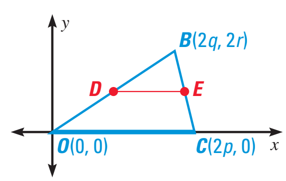
$$M\left(\frac{0+k}{2}, \frac{k+0}{2}\right) = M\left(\frac{k}{2}, \frac{k}{2}\right)$$

EXAMPLE 5 Prove the Midsegment Theorem

Write a coordinate proof of the Midsegment Theorem for one midsegment.

GIVEN ▶ $\overline{DE}$ is a midsegment of $\triangle OBC$.

PROVE ▶ $\overline{DE} \parallel \overline{OC}$ and $DE = \frac{1}{2}OC$



Solution

WRITE PROOFS
You can often assign coordinates in several ways, so choose a way that makes computation easier. In Example 5, you can avoid fractions by using $2p$, $2q$, and $2r$.

STEP 1 **Place** $\triangle OBC$ and assign coordinates. Because you are finding midpoints, use $2p$, $2q$, and $2r$. Then find the coordinates of D and E.

$$D\left(\frac{2q+0}{2}, \frac{2r+0}{2}\right) = D(q, r) \qquad E\left(\frac{2q+2p}{2}, \frac{2r+0}{2}\right) = E(q+p, r)$$

STEP 2 **Prove** $\overline{DE} \parallel \overline{OC}$. The y-coordinates of D and E are the same, so $\overline{DE}$ has a slope of 0. $\overline{OC}$ is on the x-axis, so its slope is 0.

▶ Because their slopes are the same, $\overline{DE} \parallel \overline{OC}$.

STEP 3 **Prove** $DE = \frac{1}{2}OC$. Use the Ruler Postulate to find $\overline{DE}$ and $\overline{OC}$.

$$DE = |(q+p) - q| = p \qquad OC = |2p - 0| = 2p$$

▶ So, the length of $\overline{DE}$ is half the length of $\overline{OC}$.

✓ GUIDED PRACTICE for Examples 4 and 5

7. In Example 5, find the coordinates of F, the midpoint of $\overline{OC}$. Then show that $\overline{EF} \parallel \overline{OB}$. **See margin.**

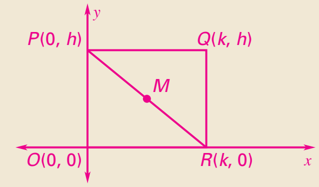
8. Graph the points $O(0, 0)$, $H(m, n)$, and $J(m, 0)$. Is $\triangle OHJ$ a right triangle? Find the side lengths and the coordinates of the midpoint of each side. **See margin.**

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 3**. This activity is also part of **Power Presentations**.

Extra Example 4

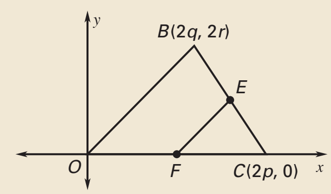
Place a rectangle in a coordinate plane. Then find the length of a diagonal and the coordinates of the midpoint M of the diagonal.



$PR = \sqrt{k^2 + h^2}$; $M\left(\frac{k}{2}, \frac{h}{2}\right)$

Extra Example 5

Write a coordinate proof of the Midsegment Theorem for the midsegment parallel to $\overline{OB}$.



Given: $\overline{FE}$ is a midsegment.

Prove: $\overline{FE} \parallel \overline{OB}$ and $FE = \frac{1}{2}OB$

The midpoints are $E(q + p, r)$ and $F = F(p, 0)$. The slope of both $\overline{FE}$ and $\overline{OB}$ is $\frac{r}{q}$ so $\overline{FE} \parallel \overline{OB}$. Also, $FE = \sqrt{q^2 + r^2}$ and $OB = 2\sqrt{q^2 + r^2}$, so $FE = \frac{1}{2}OB$.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you write a coordinate proof?

- **Assign coordinates to vertices that are convenient for finding lengths.**
- **Use coordinates to find mid-points, distances, and slopes.**

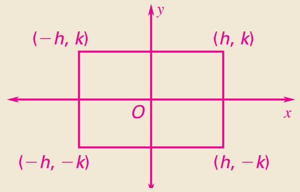
Assign convenient coordinates to vertices and use the midpoint, distance, and slope formulas to generate the proof.

Differentiated Instruction

Inclusion Some students may have difficulty grasping the idea that when variables are used to represent the coordinates of a figure, the results are true for all figures of that type. To demonstrate that this is true, have students replace the variables in **Example 5** with reasonable numbers and show that $\overline{DE}$ is half thelength of $\overline{OC}$. Have them repeat this work with other numbers to see that $\overline{DE}$ will always be half the length of $\overline{OC}$ in this type of triangle.

See also the *Differentiated Instruction Resources* for more strategies.

4.



7, 8. See Additional Answers.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1: EP for 1.3 Exs. 18–23;
Exs. 1–11
Day 2:
Exs. 12–25, 35–41

Average:
Day 1:
Exs. 1–11
Day 2:
Exs. 13–19 odd, 20, 21–27 odd, 28–32, 35–44

Advanced:
Day 1:
Exs. 1–11
Day 2:
Exs. 16–28 even, 29–46*

Block:
Exs. 1–11, 13–19 odd, 20, 21–27 odd, 28–32, 35–44

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 14, 20, 36
Average: 4, 8, 17, 23, 36
Advanced: 5, 10, 18, 22, 37

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

5.1 EXERCISES

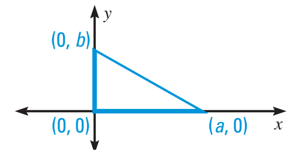
HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 9, 21, and 37
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 31, and 39

SKILL PRACTICE

A

1. **VOCABULARY** Copy and complete: In $\triangle ABC$, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$. $\overline{DE}$ is a __?__ of $\triangle ABC$. **midsegment**

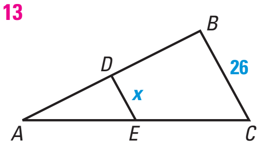
2. ★ **WRITING** *Explain* why it is convenient to place a right triangle on the grid as shown when writing a coordinate proof. How might you want to relabel the coordinates of the vertices if the proof involves midpoints? ***Sample answer:*** **The vertex of the right triangle is located at the origin and the other two vertices are located on the x-axis and y-axis which limits the number of variables need to label them; label the vertex on the x-axis $(2a, 0)$ and the vertex on the y-axis $(0, 2b)$.**



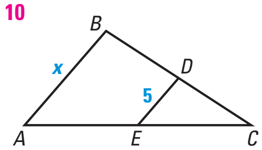
EXAMPLES 1 and 2 for Exs. 3–11

FINDING LENGTHS $\overline{DE}$ **is a midsegment of** $\triangle ABC$**. Find the value of** x**.**

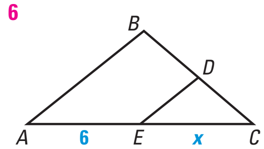
3. **13**



4. **10**



5. **6**



USING THE MIDSEGMENT THEOREM **In** $\triangle XYZ$, $\overline{XJ} \cong \overline{JY}$, $\overline{YL} \cong \overline{LZ}$, **and** $\overline{XK} \cong \overline{KZ}$. **Copy and complete the statement.**

6. $\overline{JK} \parallel$ __?__ $\overline{YZ}$

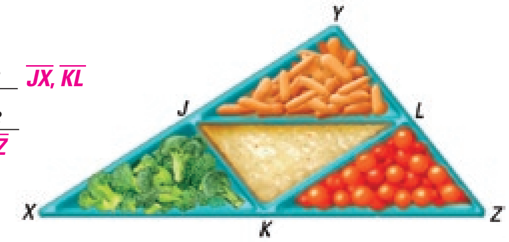
7. $\overline{JL} \parallel$ __?__ $\overline{XZ}$

8. $\overline{XY} \parallel$ __?__ $\overline{KL}$

9. $\overline{YJ} \cong$ __?__ $\cong$ __?__ $\overline{JX}, \overline{KL}$

10. $\overline{JL} \cong$ __?__ $\cong$ __?__ $\overline{XK}, \overline{KZ}$

11. $\overline{JK} \cong$ __?__ $\cong$ __?__ $\overline{YL}, \overline{LZ}$



EXAMPLE 3 for Exs. 12–19

PLACING FIGURES **Place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex.** **Sample answers are given.**

12. Right triangle: leg lengths are 3 units and 2 units **(0, 0), (3, 0), (0, 2)**

13. Isosceles right triangle: leg length is 7 units **(0, 0), (7, 0), (0, 7)**

14. Square: side length is 3 units **(0, 0), (3, 0), (3, 3), (0, 3)**

15. Scalene triangle: one side length is $2m$ **(0, 0), (2m, 0), (a, b)**

16. Rectangle: length is a and width is b **(0, 0), (a, 0), (a, b), (0, b)**

17. Square: side length is s **(0, 0), (s, 0), (s, s), (0, s)**

18. Isosceles right triangle: leg length is p **(0, 0), (p, 0), (0, p)**

19. Right triangle: leg lengths are r and s **(0, 0), (r, 0), (0, s)**

EXAMPLES 4 and 5 for Exs. 20–23

20. **COMPARING METHODS** Find the length of the hypotenuse in Exercise 19. Then place the triangle another way and use the new coordinates to find the length of the hypotenuse. Do you get the same result? $\sqrt{r^2 + s^2}$**; yes**

B

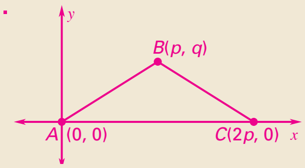
APPLYING VARIABLE COORDINATES **Sketch** $\triangle ABC$**. Find the length and the slope of each side. Then find the coordinates of each midpoint. Is** $\triangle ABC$ **a right triangle? Is it isosceles?** ***Explain.*** **(Assume all variables are positive,** $p \neq q$**, and** $m \neq n$**.)** **21–23. See margin.**

21. $A(0, 0)$, $B(p, q)$, $C(2p, 0)$

22. $A(0, 0)$, $B(h, h)$, $C(2h, 0)$

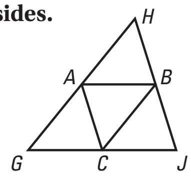
23. $A(0, n)$, $B(m, n)$, $C(m, 0)$

21.



$AB = \sqrt{p^2 + q^2}, \frac{q}{p}, \left(\frac{p}{2}, \frac{q}{2}\right)$; $BC = \sqrt{p^2 + q^2}, -\frac{q}{p}, \left(\frac{3p}{2}, \frac{q}{2}\right)$;
$CA = 2p, 0, (p, 0)$; **no; yes; it's not a right triangle because none of the slopes are negative reciprocals, and it is isosceles because two of the sides are the same length.**

ALGEBRA **Use $\triangle GHJ$, where A, B, and C are midpoints of the sides.**

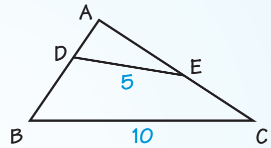


24. If $AB = 3x + 8$ and $GJ = 2x + 24$, what is AB? **14**

25. If $AC = 3y - 5$ and $HJ = 4y + 2$, what is HB? **13**

26. If $GH = 7z - 1$ and $BC = 4z - 3$, what is GH? **34**

27. **ERROR ANALYSIS** *Explain* why the conclusion is incorrect.



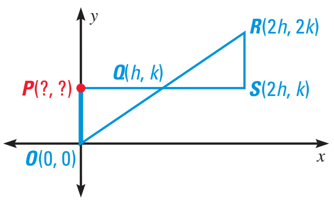
$DE = \frac{1}{2}BC$, so by the Midsegment Theorem $\overline{AD} \cong \overline{DB}$ and $\overline{AE} \cong \overline{EC}$.

You don't know that $\overline{DE}$ and $\overline{BC}$ are parallel.

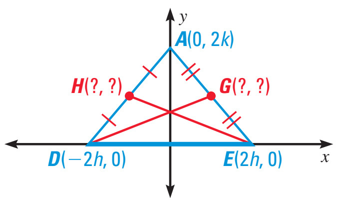
28. **FINDING PERIMETER** The midpoints of the three sides of a triangle are $P(2, 0)$, $Q(7, 12)$, and $R(16, 0)$. Find the length of each midsegment and the perimeter of $\triangle PQR$. Then find the perimeter of the original triangle.
$PQ = 13$, $PR = 14$, $QR = 15$, perimeter $= 42$; 84

APPLYING VARIABLE COORDINATES **Find the coordinates of the red point(s) in the figure. Then show that the given statement is true.** **29, 30. See margin.**

29. $\triangle OPQ \cong \triangle RSQ$



30. slope of $\overline{HE} = -(\text{slope of } \overline{DG})$



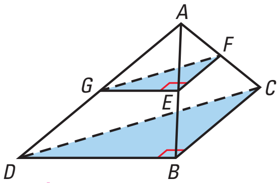
31. ★ **MULTIPLE CHOICE** A rectangle with side lengths $3h$ and k has a vertex at $(-h, k)$. Which point *cannot* be a vertex of the rectangle? **A**

(A) (h, k) (B) $(-h, 0)$ (C) $(2h, 0)$ (D) $(2h, k)$

32. **RECONSTRUCTING A TRIANGLE** The points $T(2, 1)$, $U(4, 5)$, and $V(7, 4)$ are the midpoints of the sides of a triangle. Graph the three midsegments. Then show how to use your graph and the properties of midsegments to draw the original triangle. Give the coordinates of each vertex.
See margin for art; $(-1, 2)$, $(5, 0)$, $(9, 8)$.

C

33. **3-D FIGURES** Points A, B, C, and D are the vertices of a *tetrahedron* (a solid bounded by four triangles). $\overline{EF}$ is a midsegment of $\triangle ABC$, $\overline{GE}$ is a midsegment of $\triangle ABD$, and $\overline{FG}$ is a midsegment of $\triangle ACD$.



Show that Area of $\triangle EFG = \frac{1}{4} \cdot$ Area of $\triangle BCD$. **$GE = \frac{1}{2}DB$, $EF = \frac{1}{2}BC$, area of $\triangle EFG = \frac{1}{2}\left[\frac{1}{2}DB\left(\frac{1}{2}BC\right)\right] = \frac{1}{8}(DB)(BC)$, area of $\triangle BCD = \frac{1}{2}(DB)(BC)$.**

34. **CHALLENGE** In $\triangle PQR$, the midpoint of $\overline{PQ}$ is $K(4, 12)$, the midpoint of $\overline{QR}$ is $L(5, 15)$, and the midpoint of $\overline{PR}$ is $M(6.4, 10.8)$. Show how to find the vertices of $\triangle PQR$. *Compare* your work for this exercise with your work for Exercise 32. How were your methods different? **See margin.**

29. $(0, k)$. *Sample answer:* Since $\triangle OPQ$ and $\triangle RSQ$ are right triangles with $\overline{OP} \cong \overline{RS}$ and $\overline{PQ} \cong \overline{SQ}$, the triangles are congruent by SAS.

30. $H(-h, k)$, $G(h, k)$; slope of $HE = -\frac{k}{3h}$ and the slope of $\overline{DG} = \frac{k}{3h}$.

34. Find the slope of each midsegment which gives you the slope of each side of the triangles. Use the slope and the known point on each side to find the equation of the line containing each side. Solve the three systems to find the vertices of the triangle; in Exercise 32, the vertices were found graphically and in this exercise, the vertices were found algebraically.

Study Strategy

Exercises 21–23 Students may attempt to do these exercises haphazardly. Suggest that they use a system to keep all the information organized. For example, they could first find all the lengths, then find all the slopes, and then find all the midpoints.

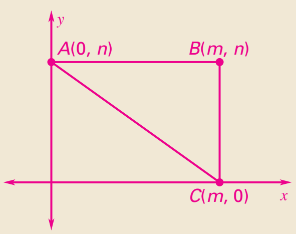
Teaching Strategy

Exercise 32 To reconstruct a triangle from the midpoints, suggest that students use the fact that midsegments are parallel to the third sides of the triangle. Have students draw lines parallel to each midsegment through the third vertex, and use the intersections of pairs of lines to find the vertices of the original triangle.

Vocabulary

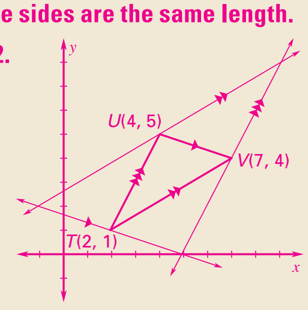
Exercise 33 A tetrahedron is a solid figure with four faces, each of which is a triangle.

23.

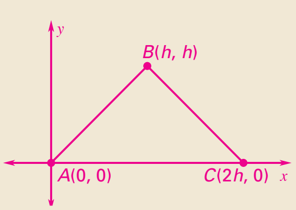


$AB = m$, 0, $\left(\frac{m}{2}, n\right)$; $BC = n$, undefined, $\left(m, \frac{n}{2}\right)$; $CA = \sqrt{m^2 + n^2}$, $-\frac{n}{m}$, $\left(\frac{m}{2}, \frac{n}{2}\right)$; yes; no; one side is vertical and one side is horizontal thus the triangle is a right triangle. It is not isosceles since none of the sides are the same length.

32.



22.



$AB = h\sqrt{2}$, 1, $\left(\frac{h}{2}, \frac{h}{2}\right)$; $BC = h\sqrt{2}$, -1, $\left(\frac{3h}{2}, \frac{h}{2}\right)$; $CA = 2h$, 0, $(h, 0)$; yes; yes; it is a right triangle and an isosceles triangle since two sides are both congruent and perpendicular.

Teaching Strategy

Exercise 42 Use this exercise to point out how using coordinates may make a proof easier. Ask students to try to plan the proof without coordinates. They should see that while it is easy to show that $CB = CD$, it is not easy to show both are equal to CA.

Mathematical Reasoning

Exercise 43 This design consists of self-similar triangles, or triangles that are similar to those in previous stages of the design. Students will study self-similarity and fractals in Chapter 6.

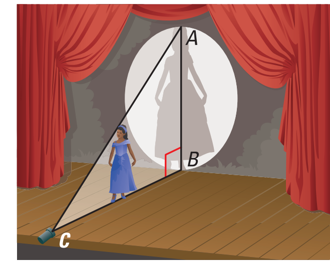
41. ***Sample answer:*** **The coordinates of D are (q, r) and the coordinates of F are $(p, 0)$ since $\left(\frac{2p+0}{2}, \frac{0+0}{2}\right) = (p, 0)$. The slope of $\overline{DF}$ is $\frac{r-0}{q-p} = \frac{r}{q-p}$ and the slope of $\overline{BC}$ is $\frac{2r-0}{2q-2p} = \frac{r}{q-p}$, so $\overline{DF} \parallel \overline{BC}$. $DF = \sqrt{(p-q)^2 + r^2}$ and $BC = \sqrt{(2q-2p)^2 + (2r)^2} = 2\sqrt{(p-q)^2 + r^2}$ making $DF = \frac{1}{2}BC$.**

42. ***Sample answer:*** **Let $A(0, 0)$, $B(0, p)$, and $D(q, 0)$ be the vertices of $\triangle ABD$. Since C is the midpoint of $\overline{BD}$, its coordinates are $\left(\frac{q}{2}, \frac{p}{2}\right)$. $AC = \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{2}\right)^2} = \frac{\sqrt{p^2+q^2}}{2}$, $BC = \sqrt{\left(\frac{q}{2} - 0\right)^2 + \left(p - \frac{p}{2}\right)^2} = \frac{\sqrt{p^2+q^2}}{2}$, and $DC = \sqrt{\left(\frac{q}{2} - q\right)^2 + \left(\frac{p}{2} - 0\right)^2} = \frac{\sqrt{p^2+q^2}}{2}$, so $AC = BC = DC$.**

PROBLEM SOLVING

A

35. FLOODLIGHTS A floodlight on the edge of the stage shines upward onto the backdrop as shown. Constance is 5 feet tall. She stands halfway between the light and the backdrop, and the top of her head is at the midpoint of $\overline{AC}$. The edge of the light just reaches the top of her head. How tall is her shadow? **10 ft**



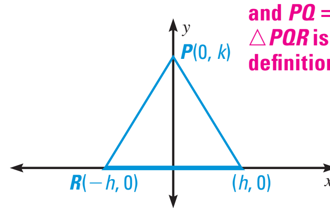
EXAMPLE 5
for Exs. 36–37

COORDINATE PROOF **Write a coordinate proof.**

36. GIVEN ▶ $P(0, k)$, $Q(h, 0)$, $R(-h, 0)$

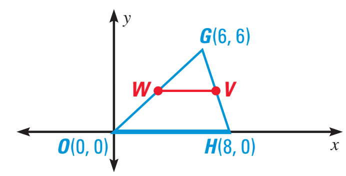
PROVE ▶ $\triangle PQR$ is isosceles.

Since $PR = \sqrt{h^2 + k^2}$ and $PQ = \sqrt{h^2 + k^2}$, $\triangle PQR$ is isosceles by definition.



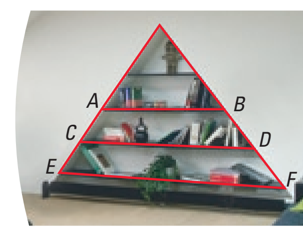
37. GIVEN ▶ $O(0, 0)$, $G(6, 6)$, $H(8, 0)$, $\overline{WV}$ is a midsegment.

PROVE ▶ $\overline{WV} \parallel \overline{OH}$ and $WV = \frac{1}{2}OH$



37. The coordinates of W are $(3, 3)$ and the coordinates of V are $(7, 3)$. The slope of $\overline{WV}$ is 0 and the slope of $\overline{OH}$ is 0 making $\overline{WV} \parallel \overline{OH}$. $WV = 4$ and $OH = 8$ thus $WV = \frac{1}{2}OH$.

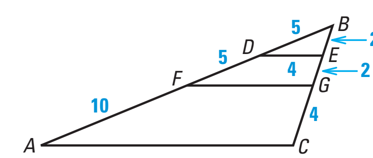
38. CARPENTRY In the set of shelves shown, the third shelf, labeled $\overline{CD}$, is closer to the bottom shelf, $\overline{EF}$, than midsegment $\overline{AB}$ is. If $\overline{EF}$ is 8 feet long, is it possible for $\overline{CD}$ to be 3 feet long? 4 feet long? 6 feet long? 8 feet long? *Explain*. **No, no, yes, no; $AB = 4$ feet since it is half the length of $\overline{EF}$. The length of $\overline{CD}$ must be greater than 4 feet but less than 8 feet.**



B

39. ★ SHORT RESPONSE Use the information in the diagram at the right. What is the length of side $\overline{AC}$ of $\triangle ABC$? *Explain* your reasoning.

16. *Sample answer:* DE is half the length of $\overline{FG}$ which makes $FG = 8$. FG is half the length of $\overline{AC}$ which makes $AC = 16$.

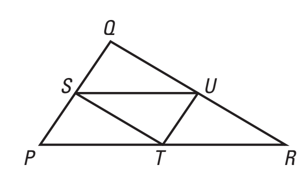


40. PLANNING FOR PROOF Copy and complete the plan for proof.

GIVEN ▶ $\overline{ST}$, $\overline{TU}$, and $\overline{SU}$ are midsegments of $\triangle PQR$.

PROVE ▶ $\triangle PST \cong \triangle SQU$

Use __?__ to show that $\overline{PS} \cong \overline{SQ}$. Use __?__ to show that $\angle QSU \cong \angle SPT$. Use __?__ to show that $\angle$ __?__ $\cong \angle$ __?__. Use __?__ to show that $\triangle PST \cong \triangle SQU$. **Definition of midsegment, Midsegment Theorem and Corresponding Angles Postulate, Midsegment Theorem and Corresponding Angles Postulate, PST, SQU, ASA**



41. PROVING THEOREM 5.1 Use the figure in Example 5. Draw the midpoint F of $\overline{OC}$. Prove that $\overline{DF}$ is parallel to $\overline{BC}$ and $DF = \frac{1}{2}BC$. **See margin.**

○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

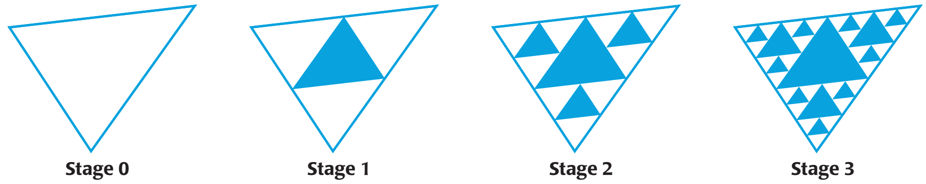
302

42. **COORDINATE PROOF** Write a coordinate proof. **See margin.**

GIVEN ▶ $\triangle ABD$ is a right triangle, with the right angle at vertex A. Point C is the midpoint of hypotenuse BD.

PROVE ▶ Point C is the same distance from each vertex of $\triangle ABD$.

43. **MULTI-STEP PROBLEM** To create the design below, shade the triangle formed by the three midsegments of a triangle. Then repeat the process for each unshaded triangle. Let the perimeter of the original triangle be 1.



a. What is the perimeter of the triangle that is shaded in Stage 1? $\frac{1}{2}$

b. What is the total perimeter of all the shaded triangles in Stage 2? $\frac{5}{4}$

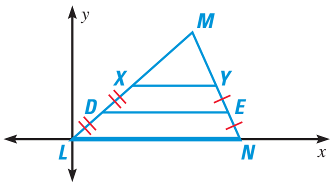
c. What is the total perimeter of all the shaded triangles in Stage 3? $\frac{19}{8}$

RIGHT ISOSCELES TRIANGLES **In Exercises 44 and 45, write a coordinate proof.** **44–46. See margin.**

44. Any right isosceles triangle can be subdivided into a pair of congruent right isosceles triangles. (*Hint:* Draw the segment from the right angle to the midpoint of the hypotenuse.)

C 45. Any two congruent right isosceles triangles can be combined to form a single right isosceles triangle.

46. **CHALLENGE** XY is a midsegment of $\triangle LMN$. Suppose $\overline{DE}$ is called a "quarter-segment" of $\triangle LMN$. What do you think an "eighth-segment" would be? Make a conjecture about the properties of a quarter-segment and of an eighth-segment. Use variable coordinates to verify your conjectures.

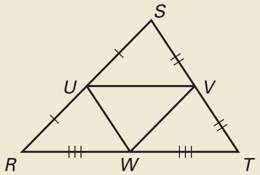


5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

Use the figure below for Exercises 1–4.



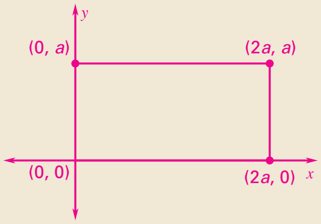
1. If $UV = 13$, find RT. **26**

2. If $ST = 20$, find UW. **10**

3. If the perimeter of $\triangle RST =$ 68 inches, find the perimeter of $\triangle UVW$. **34 in.**

4. If $VW = 2x - 4$ and $RS = 3x - 3$, what is VW? **6**

5. Place a rectangle in a coordinate plane so its vertical side has length a and its horizontal side has width $2a$. Label the coordinates of each vertex.



Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

44–46. See Additional Answers.

Alternative Strategy

Example 4 in this lesson can be solving by placing the isosceles right triangle so that the hypotenuse is on the x-axis and the right-angle vertex is on the y-axis. This is useful because the length of the hypotenuse can be found easily.

Avoiding Common Errors

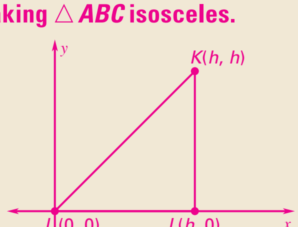
Students may not understand how the coordinates of A and B are found in the problem. Given that the hypotenuse has length $2h$ and the midpoint of the hypotenuse is (0, 0), you can conclude that the coordinates of A and B are labeled correctly.

Mathematical Reasoning

Students can use this reasoning to show that the third vertex is $C(0, h)$: In isosceles right triangle ABC, the y-axis is perpendicular to $\overline{AB}$ and $m\angle A = 45°$, so $\triangle AOC$ is an isosceles right triangle. Since $OA = h$, then $OC = h$.

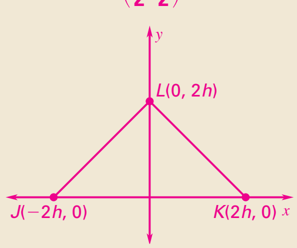
1. The slopes of $\overline{AC}$ and $\overline{BC}$ are negative reciprocals of each other, so $\overline{AC} \perp \overline{BC}$ making $\angle C$ a right angle; $AC = h\sqrt{2}$ and $BC = h\sqrt{2}$ making $\triangle ABC$ isosceles.

3a.



$JL = LK = h$ and $\overline{JL}$ is a horizontal line and $\overline{LK}$ is a vertical line, so $\overline{JL} \perp \overline{LK}$; $h\sqrt{2}$, $\left(\frac{h}{2}, \frac{h}{2}\right)$.

3b.



$JL = LK = 2h\sqrt{2}$ and the slope of $\overline{JL} = 1$ and the slope of $\overline{LK} = -1$, so $\overline{JL} \perp \overline{LK}$; $4h$, (0, 0).

PROBLEM SOLVING WORKSHOP
LESSON 5.1

Using ALTERNATIVE METHODS

Another Way to Solve Example 4

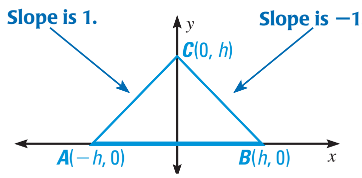
MULTIPLE REPRESENTATIONS When you write a coordinate proof, you often have several options for how to place the figure in the coordinate plane and how to assign variables.

PROBLEM

Place an isosceles right triangle in a coordinate plane. Then find the length of the hypotenuse and the coordinates of its midpoint M.

METHOD

Placing Hypotenuse on an Axis Place the triangle with point C at $(0, h)$ on the y-axis and the hypotenuse $\overline{AB}$ on the x-axis. To make $\angle ACB$ be a right angle, position A and B so that legs $\overline{CA}$ and $\overline{CB}$ have slopes of 1 and −1.



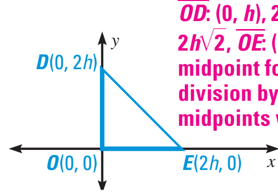
Length of hypotenuse $= 2h$

$$M = \left(\frac{-h + h}{2}, \frac{0 + 0}{2}\right) = (0, 0)$$

PRACTICE

1. **VERIFYING TRIANGLE PROPERTIES** Verify that $\angle C$ above is a right angle. Verify that $\triangle ABC$ is isosceles by showing $AC = BC$. **See margin.**

2. **MULTIPLES OF 2** Find the midpoint and length of each side using the placement below. What is the advantage of using $2h$ instead of h for the leg lengths? **$\overline{OD}$: (0, h), $2h$, $\overline{DE}$: (h, h), $2h\sqrt{2}$, $\overline{OE}$: (h, 0), $2h$; the midpoint formula requires division by 2 which results in midpoints without fractions.**

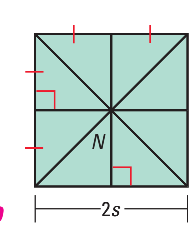


3. **OTHER ALTERNATIVES** Graph $\triangle JKL$ and verify that it is an isosceles right triangle. Then find the length and midpoint of $\overline{JK}$. **a, b. See margin.**
   a. $J(0, 0)$, $K(h, h)$, $L(h, 0)$
   b. $J(-2h, 0)$, $K(2h, 0)$, $L(0, 2h)$

4. **CHOOSE** Suppose you need to place a right isosceles triangle on a coordinate grid and assign variable coordinates. You know you will need to find all three side lengths and all three midpoints. How would you place the triangle? *Explain* your reasoning. **See margin.**

5. **RECTANGLES** Place rectangle $PQRS$ with length m and width n in the coordinate plane. Draw $\overline{PR}$ and $\overline{QS}$ connecting opposite corners of the rectangle. Then use coordinates to show that $\overline{PR} \cong \overline{QS}$. **See margin.**

6. **PARK** A square park has paths as shown. Use coordinates to determine whether a snack cart at point N is the same distance from each corner. ***Sample answer:* Square $ABCD$ with vertices at $A(0, 0)$, $B(0, 2s)$, $C(2s, 2s)$, $D(2s, 0)$, and center $N(s, s)$. $AN = BN = CN = DN = s\sqrt{2}$.**



4. *Sample answer:* Place the coordinates of the vertices at $(-2a, 0)$, $(0, 2a)$, and $(2a, 0)$ with the right angle at $(0, 2a)$; since the midpoint formula requires division by 2 the resulting points will be $(-a, a)$, (a, a), and $(0, 0)$.

5. *Sample answer:* $PQRS$ with $P(0, 0)$, $Q(0, m)$, $R(n, m)$, and $S(n, 0)$; $PR = QS = \sqrt{m^2 + n^2}$ making $\overline{PR} \cong \overline{QS}$.

5.2 Use Perpendicular Bisectors

Before You used segment bisectors and perpendicular lines.

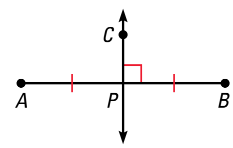
Now You will use perpendicular bisectors to solve problems.

Why? So you can solve a problem in archaeology, as in Ex. 28.

Key Vocabulary
- **perpendicular bisector**
- **equidistant**
- **concurrent**
- **point of concurrency**
- **circumcenter**

A segment bisector intersects a segment at its midpoint. A segment, ray, line, or plane that is perpendicular to a segment at its midpoint is called a **perpendicular bisector**.

A point is **equidistant** from two figures if the point is the *same distance* from each figure. Points on the perpendicular bisector of a segment are equidistant from the segment's endpoints.



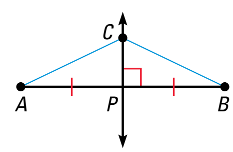
$\overleftrightarrow{CP}$ is a $\perp$ bisector of $\overline{AB}$.

THEOREMS *For Your Notebook*

THEOREM 5.2 Perpendicular Bisector Theorem

In a plane, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

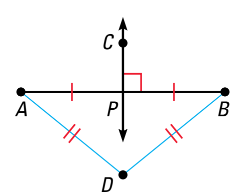
If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA = CB$.



THEOREM 5.3 Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

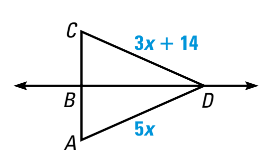
If $DA = DB$, then D lies on the $\perp$ bisector of $\overline{AB}$.



EXAMPLE 1 Use the Perpendicular Bisector Theorem

xy ALGEBRA $\overleftrightarrow{BD}$ **is the perpendicular bisector of** $\overline{AC}$. **Find** AD.

$AD = CD$	Perpendicular Bisector Theorem
$5x = 3x + 14$	Substitute.
$x = 7$	Solve for x.



▶ $AD = 5x = 5(7) = 35.$

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. Solve $3x = 8x - 15$. **3**

2. Solve $6x + 3 = 8x - 14$. **8.5**

3. If M is the midpoint of $\overline{AB}$, $AM = 5x - 2$, and $BM = 3x + 6$, find AB. **36**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 1 block

• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How do you find the point of concurrency of the perpendicular bisectors of the sides of a triangle? **Tell students they will learn how to answer this question by drawing the three perpendicular bisectors and finding their common point of intersection.**

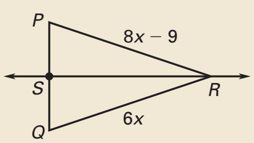
Motivating the Lesson

Have each student draw a kite and then list the features of their kite. Ask students whether, in their drawing, there is a segment that is the perpendicular bisector of another segment. Tell students that in this lesson they will explore perpendicular bisectors of segments.

3 TEACH

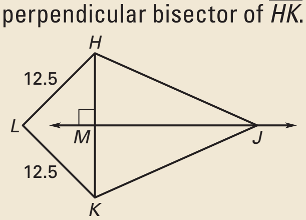
Extra Example 1

In the diagram, $\overleftrightarrow{RS}$ is the perpendicular bisector of $\overline{PQ}$. Find PR. **27**



Extra Example 2

In the diagram, $\overleftrightarrow{JM}$ is the perpendicular bisector of $\overline{HK}$.



a. Which lengths in the diagram are equal? ***$HM = KM$; $HJ = KJ$; $HL = KL$***

b. Is L on $\overleftrightarrow{JM}$? **yes**

Key Question to Ask for Example 2

- If T is another point on $\overleftrightarrow{WX}$, is $YT = ZT$? **yes**

Animated Geometry my.hrw.com

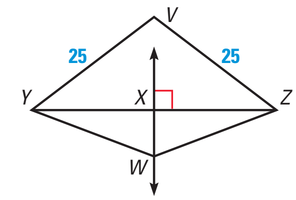
An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.

EXAMPLE 2 Use perpendicular bisectors

In the diagram, $\overleftrightarrow{WX}$ is the perpendicular bisector of $\overline{YZ}$.

a. What segment lengths in the diagram are equal?

b. Is V on $\overleftrightarrow{WX}$?



Solution

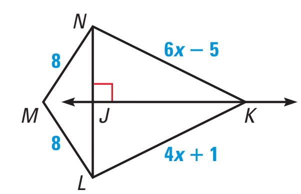
a. $\overleftrightarrow{WX}$ bisects $\overline{YZ}$, so $XY = XZ$. Because W is on the perpendicular bisector of $\overline{YZ}$, $WY = WZ$ by Theorem 5.2. The diagram shows that $VY = VZ = 25$.

b. Because $VY = VZ$, V is equidistant from Y and Z. So, by the Converse of the Perpendicular Bisector Theorem, V is on the perpendicular bisector of $\overline{YZ}$, which is $\overleftrightarrow{WX}$.

Animated Geometry at my.hrw.com

✓ GUIDED PRACTICE for Examples 1 and 2

In the diagram, $\overleftrightarrow{JK}$ is the perpendicular bisector of $\overline{NL}$.



1. What segment lengths are equal? *Explain* your reasoning.

1. $NJ = LJ$ since $\overleftrightarrow{JK}$ bisects $\overline{NL}$. $NK = LK$ by the Perpendicular Bisector Theorem and the diagram shows $ML = MN$.

2. Find NK. **13**

3. *Explain* why M is on $\overleftrightarrow{JK}$.
Since $ML = MN$, M is equidistant from N and L, so by the Converse of the Perpendicular Bisector Theorem M is on the perpendicular bisector of $\overline{NL}$ which is $\overleftrightarrow{JK}$.

ACTIVITY FOLD THE PERPENDICULAR BISECTORS OF A TRIANGLE

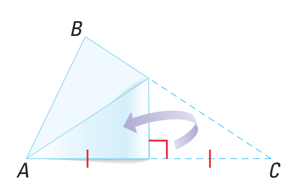
QUESTION Where do the perpendicular bisectors of a triangle meet?

Materials:
- paper
- scissors
- ruler

Follow the steps below and answer the questions about perpendicular bisectors of triangles.

STEP 1 Cut four large acute scalene triangles out of paper. Make each one different.

STEP 2 Choose one triangle. Fold it to form the perpendicular bisectors of the sides. Do the three bisectors intersect at the same point? **Yes**



STEP 3 Repeat the process for the other three triangles. Make a conjecture about the perpendicular bisectors of a triangle. **The perpendicular bisectors of a triangle intersect at one point.**

STEP 4 Choose one triangle. Label the vertices A, B, and C. Label the point of intersection of the perpendicular bisectors as P. Measure $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$. What do you observe? ***$AP = BP = CP$***

Differentiated Instruction

Below Level Students who have difficulty understanding the solution for **Example 2** may benefit from measuring the side lengths with a ruler. Ask them to verify the measurements. Then have them draw a segment 2 inches long and draw a line perpendicular to it at its midpoint. Ask them to draw triangles whose vertices are a point on the perpendicular bisector and the endpoints of the given segment and verify that the triangles are isosceles.

See also the *Differentiated Instruction Resources* for more strategies.

CONCURRENCY When three or more lines, rays, or segments intersect in the same point, they are called **concurrent** lines, rays, or segments. The point of intersection of the lines, rays, or segments is called the **point of concurrency**.

READ VOCABULARY
The perpendicular bisector of a side of a triangle can be referred to as a *perpendicular bisector of the triangle.*

Recall that the three perpendicular bisectors of a triangle are concurrent and the point of concurrency has a special property.

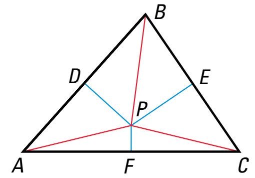
THEOREM *For Your Notebook*

THEOREM 5.4 Concurrency of Perpendicular Bisectors of a Triangle

The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA = PB = PC$.

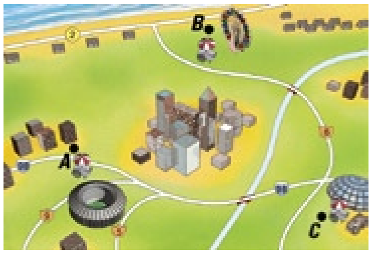
Proof: See Additional Proofs.



EXAMPLE 3 Use the concurrency of perpendicular bisectors

FROZEN YOGURT Three snack carts sell frozen yogurt from points A, B, and C outside a city. Each of the three carts is the same distance from the frozen yogurt distributor.

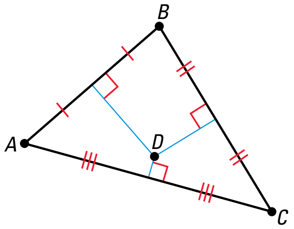
Find a location for the distributor that is equidistant from the three carts.



Solution

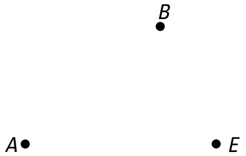
The Concurrency of Perpendicular Bisectors of a Triangle Theorem shows you that you can find a point equidistant from three points by using the perpendicular bisectors of the triangle formed by those points.

Copy the positions of points A, B, and C and connect those points to draw $\triangle ABC$. Then use a ruler and protractor to draw the three perpendicular bisectors of $\triangle ABC$. The point of concurrency D is the location of the distributor.



✓ **GUIDED PRACTICE** for Example 3

4. **WHAT IF?** Hot pretzels are sold from points A and B and also from a cart at point E. Where could the pretzel distributor be located if it is equidistant from those three points? Sketch the triangle and show the location.
Where the perpendicular bisectors of the triangle formed by *A*, *B*, and *C* intersect; see margin for art.



Activity Note

The purpose of the activity is to demonstrate that the perpendicular bisectors of the sides of any triangle meet at a single point.

Extra Example 3

Each of three forest ranger stations is the same distance from the main office. Describe how to find the location of the office. **Draw the perpendicular bisectors of the three sides. Those three lines meet at a point, and that point is the location of the office.**

Key Question to Ask for Example 3

- Is the point of concurrency always inside the triangle? **No, it can be on or outside the triangle.**

Mathematical Reasoning

You can use the Concurrency of Perpendicular Bisectors Theorem to find the center of any circle. Choose any 3 points on the circle as the vertices of a triangle and find the point of concurrency for the 3 sides of the triangle. That point is equidistant from all the points on the circle, so it is the center of the circle.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you find the point of concurrency of the perpendicular bisectors of the sides of a triangle?

- **A point on the perpendicular bisector of a segment is equidistant from the endpoints of the segments, and vice versa.**
- **The perpendicular bisectors of a triangle are concurrent.**

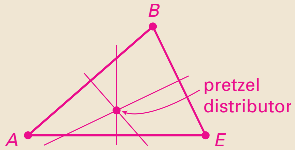
Draw each of the perpendicular bisectors and find their point of intersection.

Differentiated Instruction

Advanced Pose this question: Can you circumscribe a circle about any triangle? To answer the question, ask students to experiment with different triangles, including acute, right, and obtuse triangles, and find the point of concurrency for the perpendicular bisectors of the sides of the triangle. With that as center, use a compass to draw a circle through the vertices of the triangle.

See also the *Differentiated Instruction Resources* for more strategies.

4.



4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–15
Day 2:
Exs. 16–19, 24–28

Average:
Day 1:
Exs. 1–10, 13–15, 18, 19
Day 2:
Exs. 16, 17, 20–22, 24–30

Advanced:
Day 1:
Exs. 1–9, 13–15, 18, 19
Day 2:
Exs. 16, 17, 20–33*

Block:
Exs. 1–10, 13–22, 24–30

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 12, 16, 24, 25
Average: 6, 13, 17, 26, 28
Advanced: 8, 14, 17, 27, 28

Extra Practice

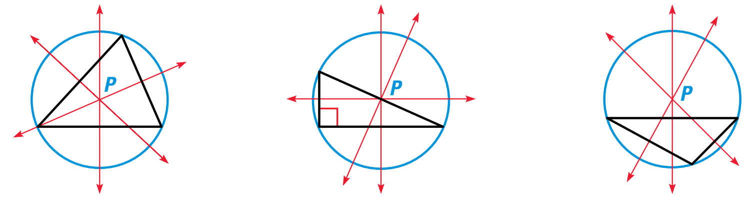
- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

READ VOCABULARY
The prefix *circum-* means "around" or "about" as in *circumference* (distance around a circle).

CIRCUMCENTER The point of concurrency of the three perpendicular bisectors of a triangle is called the **circumcenter** of the triangle. The circumcenter P is equidistant from the three vertices, so P is the center of a circle that passes through all three vertices.



Acute triangle — ***P* is inside triangle.**
Right triangle — ***P* is on triangle.**
Obtuse triangle — ***P* is outside triangle.**

As shown above, the location of P depends on the type of triangle. The circle with the center P is said to be *circumscribed* about the triangle.

5.2 EXERCISES

HOMEWORK KEY
◯ = See **WORKED-OUT SOLUTIONS** Exs. 15, 17, and 25
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 25, and 28

SKILL PRACTICE

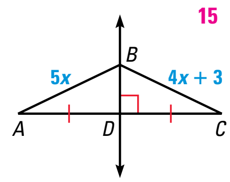
A

1. **VOCABULARY** Suppose you draw a circle with a compass. You choose three points on the circle to use as the vertices of a triangle. Copy and complete: The center of the circle is also the __?__ of the triangle. **circumcenter**

2. ★ **WRITING** Consider $\overline{AB}$. How can you *describe* the set of all points in a plane that are equidistant from A and B? **all points on the perpendicular bisector of $\overline{AB}$**

EXAMPLES 1 and 2 for Exs. 3–15

ALGEBRA Find the length of $\overline{AB}$.

3. 

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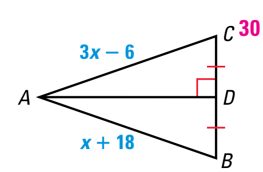
4. 

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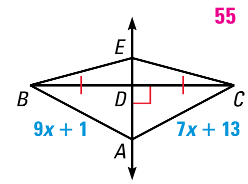
5. 

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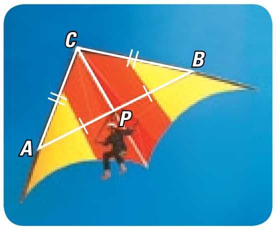
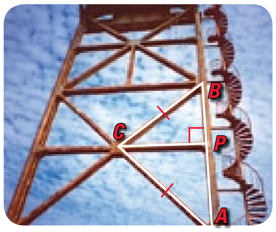
REASONING Tell whether the information in the diagram allows you to conclude that C is on the perpendicular bisector of $\overline{AB}$.

6. 

yes

7. 

yes

8. 

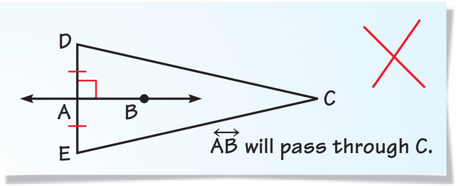
no

9. ★ **MULTIPLE CHOICE** Point P is inside $\triangle ABC$ and is equidistant from points A and B. On which of the following segments must P be located? **B**

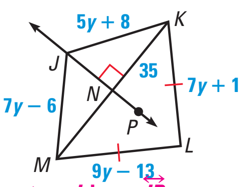
Ⓐ $\overline{AB}$ Ⓑ The perpendicular bisector of $\overline{AB}$

Ⓒ The midsegment opposite $\overline{AB}$ Ⓓ The perpendicular bisector of $\overline{AC}$

10. **ERROR ANALYSIS** *Explain* why the conclusion is not correct given the information in the diagram.
You don't know that $EC = DC$.



PERPENDICULAR BISECTORS **In Exercises 11–15, use the diagram. $\overleftrightarrow{JN}$ is the perpendicular bisector of $\overline{MK}$.**



11. Find NM. **35** 12. Find JK. **43**

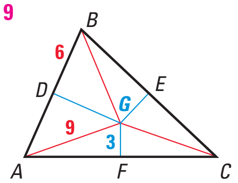
13. Find KL. **50** 14. Find ML. **50**

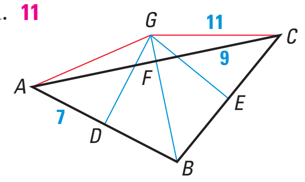
15. Is L on $\overleftrightarrow{JP}$? *Explain* your reasoning.
Yes; the Converse of the Perpendicular Bisector Theorem guarantees L is on $\overleftrightarrow{JP}$.

EXAMPLE 3 for Exs. 16–17

USING CONCURRENCY **In the diagram, the perpendicular bisectors of $\triangle ABC$ meet at point G and are shown in blue. Find the indicated measure.**

16. Find BG. **9** 17. Find GA. **11**





18. **CONSTRUCTING PERPENDICULAR BISECTORS** Construct the bisector of a segment. *Explain* why the bisector you constructed is perpendicular to the segment.

18. *Sample answer:* In the construction of the segment bisector four congruent triangles are created. In the process four pairs of congruent angles are formed which are right angles making the bisector perpendicular to the segment.

19. **CONSTRUCTION** Draw a right triangle. Use a compass and straightedge to find its circumcenter. Use a compass to draw the circumscribed circle. **See margin.**

ANALYZING STATEMENTS **Copy and complete the statement with *always*, *sometimes*, or *never*. *Justify* your answer.**

20. The circumcenter of a scalene triangle is __?__ inside the triangle.
Sometimes; a scalene triangle can be acute, right, or obtuse.

21. If the perpendicular bisector of one side of a triangle goes through the opposite vertex, then the triangle is __?__ isosceles.
Always; congruent sides are created.

22. The perpendicular bisectors of a triangle intersect at a point that is __?__ equidistant from the midpoints of the sides of the triangle.
Sometimes; consider an equilateral triangle and a scalene triangle.

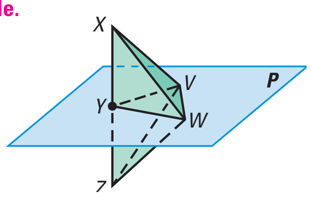
23. **CHALLENGE** Prove the statements in parts (a) – (c).

GIVEN ▶ Plane P is a perpendicular bisector of $\overline{XZ}$ at Y.

PROVE ▶ a. $\overline{XW} \cong \overline{ZW}$ **a–c. See margin.**

b. $\overline{XV} \cong \overline{ZV}$

c. $\angle VXW \cong \angle VZW$



Avoiding Common Errors

Exercises 6–10 Students may consider any line drawn from a vertex to the opposite side of the triangle a perpendicular bisector. Caution them to verify that this line is not only perpendicular (Ex. 7), but it contains the midpoint of the opposite side (Ex. 8), and is equidistant from the endpoints of a segment (Exs. 7, 10).

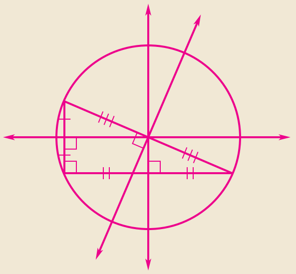
Teaching Strategy

Exercises 18–19 Ask students to verify these constructions by measuring the segment lengths. Then have them label the points in the construction and use them for their explanation.

Mathematical Reasoning

Exercise 23 Postulate 10 guarantees that if W and Y are in plane P, then $\overleftrightarrow{WY}$ is also in P. Then since X, Y, Z, and W are coplanar, it can be shown that $\overleftrightarrow{WY}$ is the perpendicular bisector of $\overline{XZ}$ in the plane of X, Y, W, and Z. Similarly, $\overleftrightarrow{VY}$ is the perpendicular bisector of $\overline{XZ}$ in the plane of X, Y, V, and Z.

19. *Sample:*



23a–c. Statements (Reasons)

1. P is a perpendicular bisector of $\overline{XZ}$ at Y, W and V lie in plane P. (Given)

2. $XW = ZW$, $XV = ZV$ (Perpendicular Bisector Theorem)

3. $\overline{XW} \cong \overline{ZW}$, $\overline{XV} \cong \overline{ZV}$ (Definition of segment congruence)

4. $\overline{WV} \cong \overline{WV}$ (Reflexive Property of Segment Congruence)

5. $\triangle VXW \cong \triangle VZW$ (SSS)

6. $\angle VXW \cong \angle VZW$ (Corr. parts of $\cong$ △ are $\cong$.)

Internet Reference

Exercise 24 For more information about cable-stayed bridges, visit www.pbs.org/wgbh/nova/bridge/meetcable.html

Exercise 28 Additional information about archaeology can be found at www.cr.nps.gov/archeology/sites/fedarch.htm

26. Statements (Reasons)

1. $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$. (Given)
2. $AP = BP$, $m\angle CPA = m\angle CPB = 90°$ (Definition of perpendicular bisector)
3. $\overline{AP} \cong \overline{BP}$ (Definition of segment congruence)
4. $\angle CPA \cong \angle CPB$ (Definition of angle congruence)
5. $\overline{CP} \cong \overline{CP}$ (Reflexive Property of Segment Congruence)
6. $\triangle CPA \cong \triangle CPB$ (SAS)
7. $\overline{CA} \cong \overline{CB}$ (Corr. parts of $\cong$ $\triangle$s are $\cong$.)
8. $CA = CB$ (Definition of segment congruence)

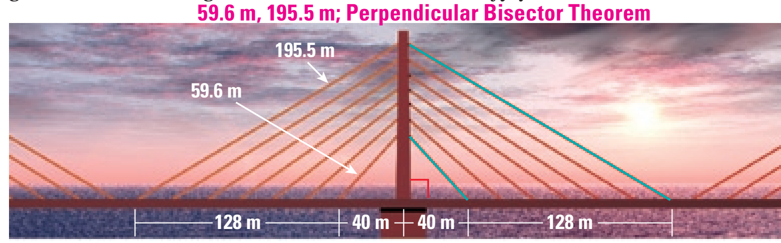
27. Statements (Reasons)

1. $CA = CB$ (Given)
2. Draw $\overleftrightarrow{PC} \perp \overline{AB}$ through point C. (Perpendicular Postulate)
3. $\overline{CA} \cong \overline{CB}$ (Definition of segment congruence)
4. $\overline{CP} \cong \overline{CP}$ (Reflexive Property of Segment Congruence)
5. $\angle CPA$ and $\angle CPB$ are right angles. (Definition of perpendicular lines)
6. $\triangle CPA$ and $\triangle CPB$ are right triangles. (Definition of right triangle)
7. $\triangle CPA \cong \triangle CPB$ (HL)
8. $\overline{PA} \cong \overline{PB}$ (Corr. parts of $\cong$ $\triangle$s are $\cong$.)
9. P is the midpoint of $\overline{AB}$. (Definition of midpoint)
10. C is on the perpendicular bisector of $\overline{AB}$. (Definition of perpendicular bisector)

PROBLEM SOLVING

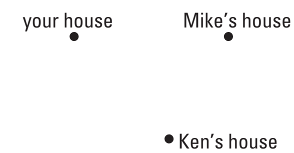
A

24. BRIDGE A cable-stayed bridge is shown below. Two cable lengths are given. Find the lengths of the blue cables. *Justify* your answer.
59.6 m, 195.5 m; Perpendicular Bisector Theorem



EXAMPLE 3 for Exs. 25, 28

25. ★ **SHORT RESPONSE** You and two friends plan to walk your dogs together. You want your meeting place to be the same distance from each person's house. *Explain* how you can use the diagram to locate the meeting place.



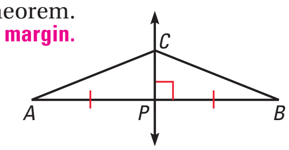
25. The Concurrency of Perpendicular Bisectors of a Triangle theorem shows you that you can find a point equidistant from three points by using the perpendicular bisectors of the sides of the triangle formed by the three points.

26. PROVING THEOREM 5.2 Prove the Perpendicular Bisector Theorem. **See margin.**

GIVEN ▶ $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$.

PROVE ▶ $CA = CB$

Plan for Proof Show that right triangles $\triangle APC$ and $\triangle BPC$ are congruent. Then show that $\overline{CA} \cong \overline{CB}$.

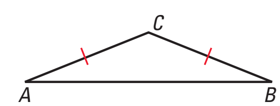


B

27. PROVING THEOREM 5.3 Prove the converse of the Perpendicular Bisector Theorem. (*Hint:* Construct a line through C perpendicular to $\overline{AB}$ at P.) **See margin.**

GIVEN ▶ $CA = CB$

PROVE ▶ C is on the perpendicular bisector of $\overline{AB}$.

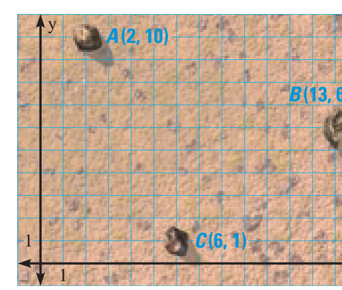


28a. Find the intersection of the perpendicular bisectors of the triangle formed by the three points.

28. ★ **EXTENDED RESPONSE** Archaeologists find three stones. They believe that the stones were once part of a circle of stones with a community firepit at its center. They mark the locations of Stones A, B, and C on a graph where distances are measured in feet.

a. *Explain* how the archaeologists can use a sketch to estimate the center of the circle of stones.

b. Copy the diagram and find the approximate coordinates of the point at which the archaeologists should look for the firepit. **approximately (7, 6.5)**



29. TECHNOLOGY Use geometry drawing software to construct $\overline{AB}$. Find the midpoint C. Draw the perpendicular bisector of $\overline{AB}$ through C. Construct a point D along the perpendicular bisector and measure $\overline{DA}$ and $\overline{DB}$. Move D along the perpendicular bisector. What theorem does this construction demonstrate? **Check students' work; Perpendicular Bisector Theorem.**

◯ = See **WORKED-OUT SOLUTIONS** in Student Resources

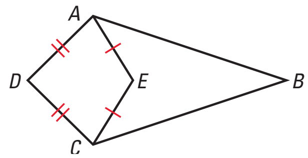
★ = **STANDARDIZED TEST PRACTICE**

30. Midpoint of the hypotenuse. *Sample answer:* Right triangle with vertices $A(2a, 0)$, $B(0, 2b)$, and $C(0, 0)$; the midpoint of $\overline{AC}$ is $(a, 0)$ and the midpoint of $\overline{BC}$ is $(0, b)$. The equations of the perpendicular bisectors of $\overline{AC}$ and $\overline{BC}$ are $x = a$ and $y = b$. These two lines intersect in the point (a, b), which is the midpoint of $\overline{AB}$.

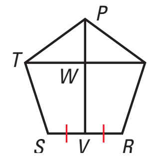
30. **COORDINATE PROOF** Where is the circumcenter located in any right triangle? Write a coordinate proof of this result. **See margin.**

C **PROOF** **Use the information in the diagram to prove the given statement.** **31, 32. See margin.**

31. $\overline{AB} \cong \overline{BC}$ if and only if D, E, and B are collinear.



32. $\overline{PV}$ is the perpendicular bisector of $\overline{TQ}$ for regular polygon $PQRST$.

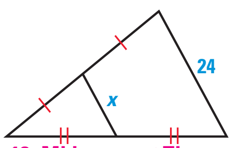


33. **CHALLENGE** The four towns on the map are building a common high school. They have agreed that the school should be an equal distance from each of the four towns. Is there a single point where they could agree to build the school? If so, find it. If not, *explain* why not. Use a diagram to *explain* your answer. **No; unless the four points determine a rectangle there is no single point to locate the school so that it is equidistant from the four towns. See margin for art.**

QUIZ

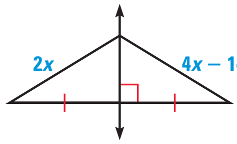
Find the value of x. Identify the theorem used to find the answer.

1.



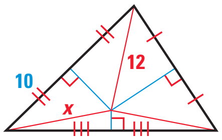
12; Midsegment Theorem

2.



7; Perpendicular Bisector Theorem

3.



3. 12; Concurrency of Perpendicular Bisectors Theorem

4. Graph the triangle with vertices $R(2a, 0)$, $S(0, 2b)$, and $T(2a, 2b)$, where a and b are positive. Find RT and ST. Then find the slope of $\overline{SR}$ and the coordinates of the midpoint of $\overline{SR}$. **$2b$, $2a$; $-\frac{b}{a}$, (a, b); see margin for art.**

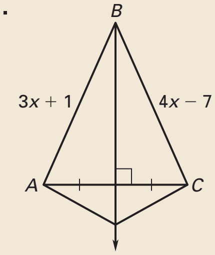
5 ASSESS AND RETEACH

Daily Homework Quiz

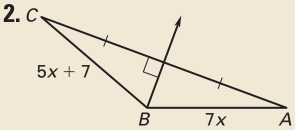
Also available online

In Exercises 1 and 2, find AB.

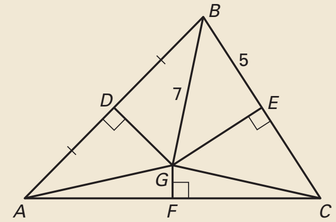
1. **25**



2. **24.5**



3. In this diagram, the perpendicular bisectors of $\triangle ABC$ meet at point G. Find EC and GC. **5; 7**



Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

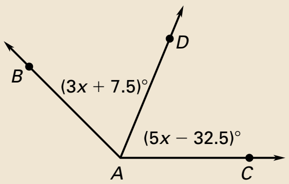
31–33, Quiz 4. See Additional Answers.

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. $\overrightarrow{AD}$ bisects $\angle BAC$. Find x. **20**



2. Solve $x^2 + 12^2 = 169$. **±5**
3. The legs of a right triangle measure 15 feet and 20 feet. Find the length of the hypotenuse. **25 ft**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day

Average: 1 day

Advanced: 1 day

Block: 0.5 block with next lesson

• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

When can you conclude that a point is on the bisector of an angle? **Tell students they will learn how to answer this question by applying the Angle Bisector Theorem and its converse.**

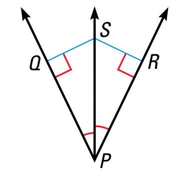
5.3 Use Angle Bisectors of Triangles

Before	You used angle bisectors to find angle relationships.
Now	You will use angle bisectors to find distance relationships.
Why?	So you can apply geometry in sports, as in Example 2.

Key Vocabulary
- **incenter**
- **angle bisector,**
- **distance from a point to a line,**

Remember that an *angle bisector* is a ray that divides an angle into two congruent adjacent angles. Remember also that the *distance from a point to a line* is the length of the perpendicular segment from the point to the line.

So, in the diagram, $\overrightarrow{PS}$ is the bisector of $\angle QPR$ and the distance from S to $\overrightarrow{PQ}$ is SQ, where $\overline{SQ} \perp \overrightarrow{PQ}$.



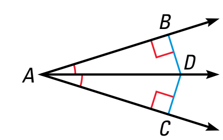
REVIEW DISTANCE
In Geometry, *distance* means the *shortest* length between two objects.

THEOREMS *For Your Notebook*

THEOREM 5.5 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

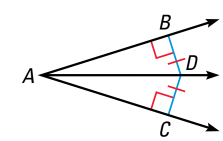
If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB = DC$.



THEOREM 5.6 Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.

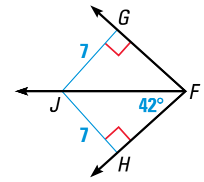


EXAMPLE 1 Use the Angle Bisector Theorems

Find the measure of $\angle GFJ$.

Solution

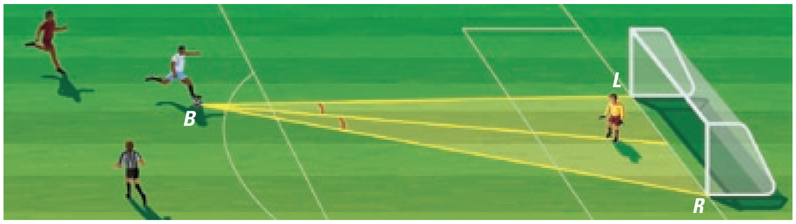
Because $\overline{JG} \perp \overrightarrow{FG}$ and $\overline{JH} \perp \overrightarrow{FH}$ and $JG = JH = 7$, $\overrightarrow{FJ}$ bisects $\angle GFH$ by the Converse of the Angle Bisector Theorem. So, $m\angle GFJ = m\angle HFJ = 42°$.



Tracy Frankel/Getty Images

EXAMPLE 2 Solve a real-world problem

SOCCER A soccer goalie's position relative to the ball and goalposts forms congruent angles, as shown. Will the goalie have to move farther to block a shot toward the right goalpost R or the left goalpost L?



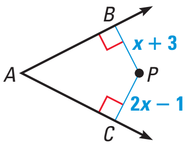
Solution

The congruent angles tell you that the goalie is on the bisector of $\angle LBR$. By the Angle Bisector Theorem, the goalie is equidistant from $\overrightarrow{BR}$ and $\overrightarrow{BL}$.

▸ So, the goalie must move the same distance to block either shot.

EXAMPLE 3 Use algebra to solve a problem

ALGEBRA **For what value of x does P lie on the bisector of $\angle A$?**



Solution

From the Converse of the Angle Bisector Theorem, you know that P lies on the bisector of $\angle A$ if P is equidistant from the sides of $\angle A$, so when $BP = CP$.

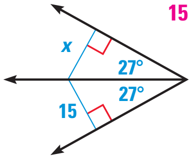
$BP = CP$	Set segment lengths equal.
$x + 3 = 2x - 1$	Substitute expressions for segment lengths.
$4 = x$	Solve for x.

▸ Point P lies on the bisector of $\angle A$ when $x = 4$.

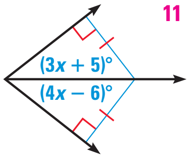
✓ GUIDED PRACTICE for Examples 1, 2, and 3

In Exercises 1–3, find the value of x.

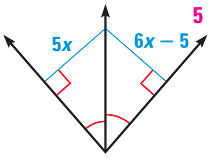
1. **15**



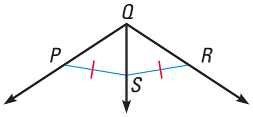
2. **11**



3. **5**



4. Do you have enough information to conclude that $\overrightarrow{QS}$ bisects $\angle PQR$? Explain.
No; you need to establish that $\overline{SR} \perp \overline{QR}$ and $\overline{SP} \perp \overline{QP}$.



Differentiated Instruction

Visual Learners Before students read the solution in **Example 3**, have them draw the angle bisector of $\angle A$ by drawing a segment from point A that goes through point P. After students have solved for x, discuss with them what must be true of the segment they drew if they substitute any other number than 4 for x. Students should realize the segment they drew does not bisect the angle if x is any number other than 4.

See also the *Differentiated Instruction Resources* for more strategies.

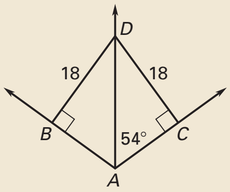
Motivating the Lesson

Show students a triangle, and ask them how they could draw circles that lie inside or on, but not outside, the triangle. Tell students that in this lesson they will learn how to draw a circle that just touches all three sides of the triangle.

3 TEACH

Extra Example 1

Find the measure of $\angle BAD$. **54°**

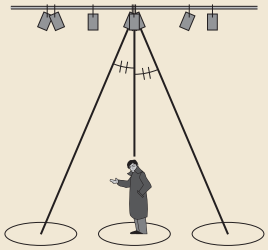


Key Question to Ask for Example 1

- If $JH \neq JG$, can you conclude that J is on the bisector of $\angle GFH$? **no**

Extra Example 2

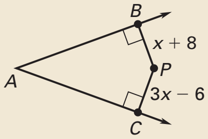
Three spotlights form two congruent angles. Is the actor closer to the spotlighted area on the right or on the left?



The actor is the same distance from both spotlighted areas.

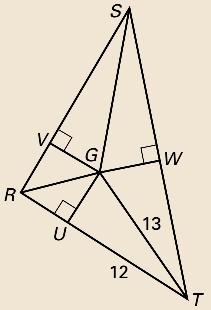
Extra Example 3

For what value of x does P lie on the bisector of $\angle A$? **7**



Extra Example 4

In the diagram, G is the incenter of $\triangle RST$. Find GW. **5**



Key Question to Ask for Example 4

- Why does $NF = ND$? **The incenter is equidistant from the three sides of the triangle.**

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 4**. This activity is also part of **Power Presentations**.

Vocabulary

Be sure students distinguish between a triangle's incenter, which is the intersection of the angle bisectors of the triangle, and its circumcenter, which is the intersection of the perpendicular bisectors of the sides.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: When can you conclude that a point is on the bisector of an angle?

- **If a point is on the bisector of an angle it is equidistant from the sides of the angle, and vice versa.**
- **The angle bisectors of a triangle intersect at a point equidistant from the triangle's sides.**

A point is on the bisector of an angle if it is in the interior of the angle and is equidistant from the sides of the angle.

READ VOCABULARY

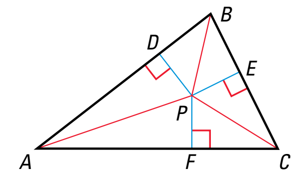
An *angle bisector of a triangle* is the bisector of an interior angle of the triangle.

THEOREM *For Your Notebook*

THEOREM 5.7 Concurrency of Angle Bisectors of a Triangle

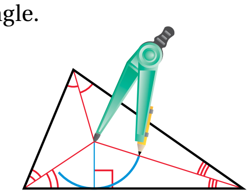
The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

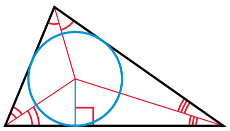
If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.



The point of concurrency of the three angle bisectors of a triangle is called the **incenter** of the triangle. The incenter always lies inside the triangle.

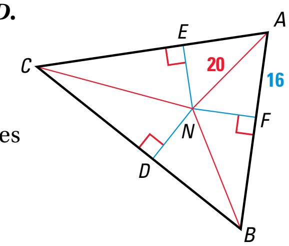
Because the incenter P is equidistant from the three sides of the triangle, a circle drawn using P as the center and the distance to one side as the radius will just touch the other two sides. The circle is said to be *inscribed* within the triangle.





EXAMPLE 4 Use the concurrency of angle bisectors

In the diagram, N is the incenter of $\triangle ABC$. Find ND.



Solution

By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter N is equidistant from the sides of $\triangle ABC$. So, to find ND, you can find NF in $\triangle NAF$. Use the Pythagorean Theorem.

$c^2 = a^2 + b^2$	**Pythagorean Theorem**
$20^2 = NF^2 + 16^2$	**Substitute known values.**
$400 = NF^2 + 256$	**Multiply.**
$144 = NF^2$	**Subtract 256 from each side.**
$12 = NF$	**Take the positive square root of each side.**

▸ Because $NF = ND$, $ND = 12$.

Animated Geometry at my.hrw.com

REVIEW QUADRATIC EQUATIONS

For help with solving a quadratic equation by taking square roots, see p. SR14. Use only the positive square root when finding a distance, as in Example 4.

✓ GUIDED PRACTICE for Example 4

5. **WHAT IF?** In Example 4, suppose you are not given AF or AN, but you are given that $BF = 12$ and $BN = 13$. Find ND. **5**

5.3 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 7, 15, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 23, 30, and 31

SKILL PRACTICE

A

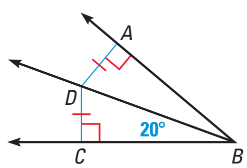
1. **VOCABULARY** Copy and complete: Point C is in the interior of $\angle ABD$. If $\angle ABC$ and $\angle DBC$ are congruent, then $\overrightarrow{BC}$ is the _?_ of $\angle ABD$. **bisector**

2. ★ **WRITING** How are perpendicular bisectors and angle bisectors of a triangle different? How are they alike? **See margin.**

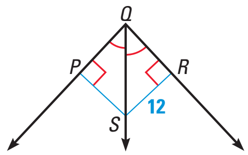
EXAMPLE 1 for Exs. 3–5

FINDING MEASURES **Use the information in the diagram to find the measure.**

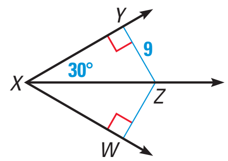
3. Find $m\angle ABD$. **20°**



4. Find PS. **12**



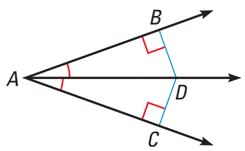
5. $m\angle YXW = 60°$. Find WZ. **9**



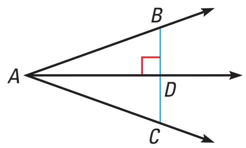
EXAMPLE 2 for Exs. 6–11

ANGLE BISECTOR THEOREM **Is $DB = DC$? Explain.** **6–8. See margin.**

6.



○7.



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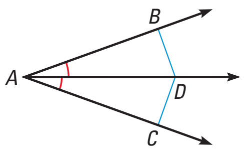
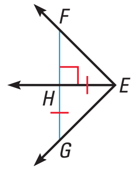


2. Perpendicular bisectors bisect line segments while angle bisectors bisect angles; both divide the segment or angle into two equal parts, and both have special points of intersection.

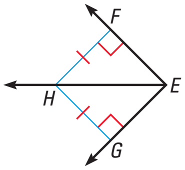
REASONING **Can you conclude that $\overrightarrow{EH}$ bisects $\angle FEG$? Explain.**

9.



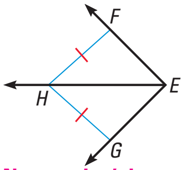
No; you don't know that $\overline{HG} \cong \overline{HF}$, $\overline{HF} \perp \overrightarrow{EF}$, or $\overline{HG} \perp \overrightarrow{EG}$.

10.



Yes; Converse of Angle Bisector Theorem

11.

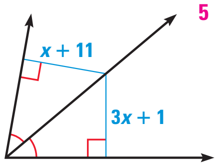


No; you don't know that $\overline{HF} \perp \overrightarrow{EF}$ or $\overline{HG} \perp \overrightarrow{EG}$.

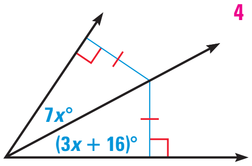
EXAMPLE 3 for Exs. 12–18

xy ALGEBRA **Find the value of x.**

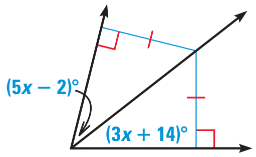
12. **5**



13. **4**

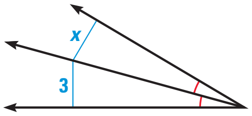


14. **8**

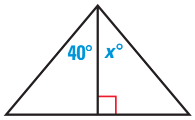


RECOGNIZING MISSING INFORMATION **Can you find the value of x? Explain.** **15–17. See margin.**

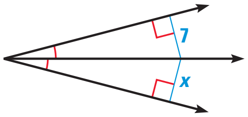
○15.



16.



17.



4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–17 odd, 18–24, 28–33

Average:
Day 1:
Exs. 1, 2–22 even, 23–26, 28–36

Advanced:
Day 1:
Exs. 1, 2, 5, 8, 11, 14, 17–38*

Block:
Exs. 1, 2–22 even, 23–26, 28–36 (with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 7, 13, 19, 28

Average: 4, 8, 14, 20, 29

Advanced: 5, 11, 17, 20, 30

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

6. Yes; $\angle BAD \cong \angle CAD$, $\overline{DB} \perp \overline{AB}$, and $\overline{DC} \perp \overline{AC}$ so by the Angle Bisector Theorem $DB = DC$.

7. No; you do not know that $\angle BAD \cong \angle CAD$.

8. No; you do not know that $\overline{DB} \perp \overline{AB}$ or $\overline{DC} \perp \overline{AC}$.

15. No; the segments with length x and 3 are not perpendicular to their respective rays.

16. No; you do not know that the perpendicular segment bisects the angle.

17. Yes; $x = 7$ using the Angle Bisector Theorem.

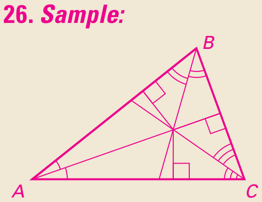
Avoiding Common Errors

Exercises 6–11 Applying the Angle Bisector Theorems can be confusing for some students. Point out that $\overline{BD}$ does not represent the distance from B to the angle bisector in Exercises 7 and 8, and $\overline{FH}$ does not represent the distance from F to the angle bisector in Exercises 9 and 11.

21. GD is not the perpendicular distance from G to $\overline{CE}$. The same is true about GF; the distance from G to each side of the triangle is the same.

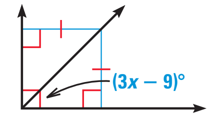
22. T is not the incenter of $\triangle UWY$. *Sample answer:* $TU = TW = TY$

26. *Sample:*



27. *Sample answer:* Since $\triangle ABC$ is a right triangle, its area is $\frac{1}{2}(AB \cdot AC)$. The area of $\triangle ABC$ is also the sum of the areas of $\triangle ABD$, $\triangle ADC$, and $\triangle DBC$. This sum is $\frac{1}{2}x(AB) + \frac{1}{2}x(AC) + \frac{1}{2}x(BC)$, or $\frac{1}{2}x(AC + AB + BC)$. Setting $\frac{1}{2}(AB \cdot AC)$ equal to $\frac{1}{2}x(AC + AB + BC)$ and solving for x gives $x = \frac{AB \cdot AC}{AC + AB + BC}$.

18. ★ **MULTIPLE CHOICE** What is the value of x in the diagram? **B**



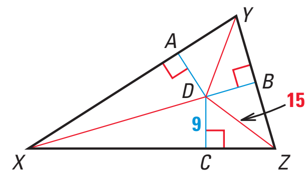
Ⓐ 13 Ⓑ 18

Ⓒ 33 Ⓓ Not enough information

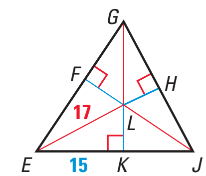
EXAMPLE 4 for Exs. 19–22

USING INCENTERS **Find the indicated measure.**

19. Point D is the incenter of $\triangle XYZ$. Find DB. **9**

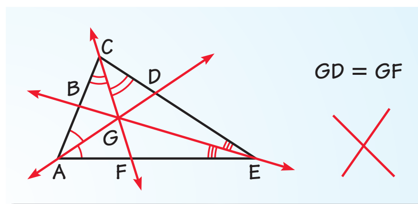


20. Point L is the incenter of $\triangle EGJ$. Find HL. **8**

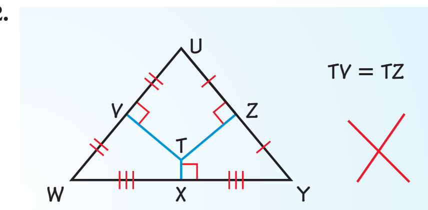


ERROR ANALYSIS ***Describe*** **the error in reasoning. Then state a correct conclusion about distances that can be deduced from the diagram.** **21, 22. See margin.**

21.



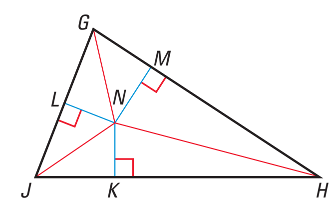
22.



B **23.** ★ **MULTIPLE CHOICE** In the diagram, N is the incenter of $\triangle GHJ$. Which statement cannot be deduced from the given information? **C**

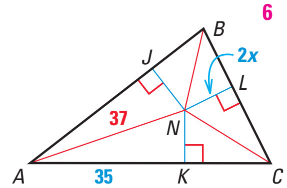
Ⓐ $\overline{NM} \cong \overline{NK}$ Ⓑ $\overline{NL} \cong \overline{NM}$

Ⓒ $\overline{NG} \cong \overline{NJ}$ Ⓓ $\overline{HK} \cong \overline{HM}$

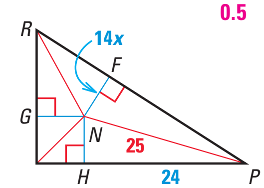


xy ALGEBRA **Find the value of x that makes N the incenter of the triangle.**

24. **6**

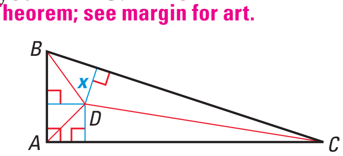


25. **0.5**



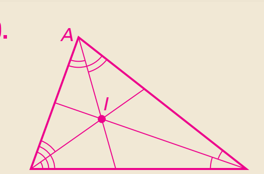
26. **CONSTRUCTION** Use a compass and a straightedge to draw $\triangle ABC$ with incenter D. Label the angle bisectors and the perpendicular segments from D to each of the sides of $\triangle ABC$. Measure each segment. What do you notice? What theorem have you verified for your $\triangle ABC$? **They all have the same length; Concurrency of Angle Bisectors of a Triangle Theorem; see margin for art.**

C **27.** **CHALLENGE** Point D is the incenter of $\triangle ABC$. Write an expression for the length x in terms of the three side lengths AB, AC, and BC. **See margin.**

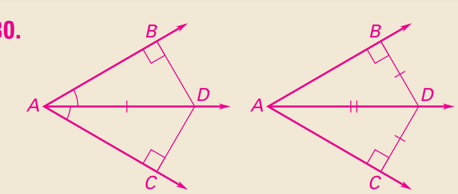


◯ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE**

29.



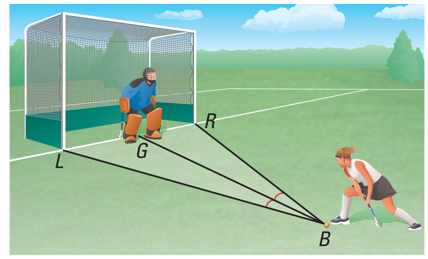
30.



PROBLEM SOLVING

EXAMPLE 2 [A] for Ex. 28

28. FIELD HOCKEY In a field hockey game, the goalkeeper is at point G and a player from the opposing team hits the ball from point B. The goal extends from left goalpost L to right goalpost R. Will the goalkeeper have to move farther to keep the ball from hitting L or R? *Explain.*



No; G is on the angle bisector of $\angle LBR$.

29. KOI POND You are constructing a fountain in a triangular koi pond. You want the fountain to be the same distance from each edge of the pond. Where should you build the fountain? *Explain* your reasoning. Use a sketch to support your answer.

At the incenter of the pond; see margin for art.

30. ★ SHORT RESPONSE What congruence postulate or theorem would you use to prove the Angle Bisector Theorem? to prove the Converse of the Angle Bisector Theorem? Use diagrams to show your reasoning. **AAS; HL; see margin for art.**

[B] **31. ★ EXTENDED RESPONSE** Suppose you are given a triangle and are asked to draw all of its perpendicular bisectors and angle bisectors.

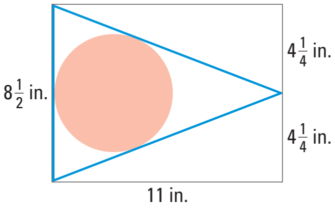
a. For what type of triangle would you need the fewest segments? What is the minimum number of segments you would need? *Explain.*

b. For what type of triangle would you need the most segments? What is the maximum number of segments you would need? *Explain.* **Scalene; 6; each angle bisector would be different than the corresponding perpendicular bisector.**

31a. Equilateral; 3; the angle bisector would also be the perpendicular bisector.

CHOOSING A METHOD In Exercises 32 and 33, tell whether you would use *perpendicular bisectors* or *angle bisectors*. Then solve the problem.

32. BANNER To make a banner, you will cut a triangle from an $8\frac{1}{2}$ inch by 11 inch sheet of white paper and paste a red circle onto it as shown. The circle should just touch each side of the triangle. Use a model to decide whether the circle's radius should be *more* or *less* than $2\frac{1}{2}$ inches. Can you cut the circle from a 5 inch by 5 inch red square? *Explain.*



Angle bisector; more; no; the diameter of the inscribed circle is greater than 5 inches.

33. CAMP A map of a camp shows a pool at (10, 20), a nature center at (16, 2), and a tennis court at (2, 4). A new circular walking path will connect the three locations. Graph the points and find the approximate center of the circle. Estimate the radius of the circle if each unit on the grid represents 10 yards. Then use the formula $C = 2\pi r$ to estimate the length of the path. **Perpendicular bisectors; (10, 10); 100 yd; about 628 yd; see margin for art.**

PROVING THEOREMS 5.5 AND 5.6 Use Exercise 30 to prove the theorem. 34, 35. See margin.

34. Angle Bisector Theorem

35. Converse of the Angle Bisector Theorem

Internet Reference

Exercise 28 For more information about field hockey, visit www.usfieldhockey.com/hockey/index.htm

Reading Strategy

Exercises 32–33 These exercises can help students distinguish between the incenter and the circumcenter of a triangle. Ask students to discuss the directions with a partner and their strategies for answering the questions.

Teaching Strategy

Exercise 38 Ask students to draw diagrams of different triangles, including obtuse ones, to help them plan their proof. Each diagram should be labeled with the same letters or variables so that the proof applies to all of them.

35. Statements (Reasons)

1. $\angle BAC$ with D in its interior, $\overline{DB} \perp \overline{AB}$, $\overline{DC} \perp \overline{AC}$, $DB = DC$. (Given)

2. $\angle ABD$ and $\angle ACD$ are right angles. (Definition of perpendicular lines)

3. $\triangle ABD$ and $\triangle ACD$ are right triangles. (Definition of right triangle)

4. $\overline{DB} \cong \overline{DC}$ (Definition of congruent segments)

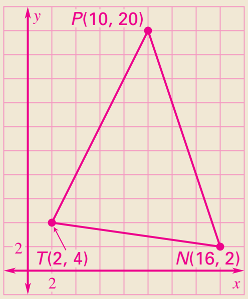
5. $\overline{AD} \cong \overline{AD}$ (Reflexive Property of Segment Congruence)

6. $\triangle ABD \cong \triangle ACD$ (HL)

7. $\angle BAD \cong \angle CAD$ (Corr. parts of $\cong$ △s are $\cong$.)

8. $\overrightarrow{AD}$ bisects $\angle BAC$. (Definition of angle bisector)

33.



34. Statements (Reasons)

1. $\angle BAC$ is bisected by $\overline{AD}$, $\overline{DB} \perp \overline{AB}$, $\overline{DC} \perp \overline{AC}$. (Given)

2. $\angle BAD \cong \angle CAD$ (Definition of angle bisector)

3. $\angle DBA$ and $\angle DCA$ are right angles. (Definition of perpendicular lines)

4. $\angle DBA \cong \angle DCA$ (Right Angles Congruence Theorem)

5. $\overline{DA} \cong \overline{DA}$ (Reflexive Property of Segment Congruence)

6. $\triangle ABD \cong \triangle ACD$ (AAS)

7. $\overline{DB} \cong \overline{DC}$ (Corr. parts of $\cong$ △s are $\cong$.)

8. $DB = DC$ (Definition of congruent segments)

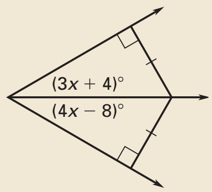
5 ASSESS AND RETEACH

Daily Homework Quiz

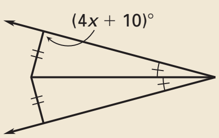
Also available online

Find the value of x.

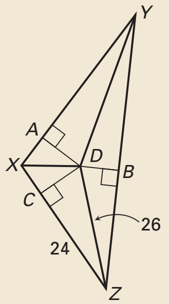
1. 12



2. 20



3. Point D is the incenter of $\triangle XYZ$. Find DB. 10



Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

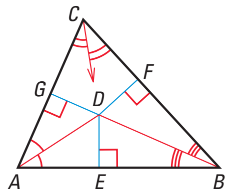
Challenge

Additional challenge is available in the Chapter Resource Book.

36, 37a. See Additional Answers.

37a. Use the Concurrency of Angle Bisectors of Triangle Theorem; if you move the circle to any other spot it will extend into the walkway; see margin for art.

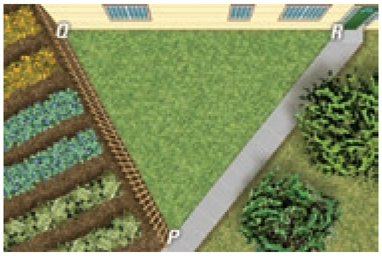
36. PROVING THEOREM 5.7 Write a proof of the Concurrency of Angle Bisectors of a Triangle Theorem. **See margin.**



GIVEN ▶ $\triangle ABC$, $\overline{AD}$ bisects $\angle CAB$, $\overline{BD}$ bisects $\angle CBA$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$, $\overline{DG} \perp \overline{CA}$

PROVE ▶ The angle bisectors intersect at D, which is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

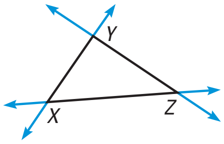
C **37. CELEBRATION** You are planning a graduation party in the triangular courtyard shown. You want to fit as large a circular tent as possible on the site without extending into the walkway.



a. Copy the triangle and show how to place the tent so that it just touches each edge. Then *explain* how you can be sure that there is no place you could fit a larger tent on the site. Use sketches to support your answer.

b. Suppose you want to fit as large a tent as possible while leaving at least one foot of space around the tent. Would you put the center of the tent in the same place as you did in part (a)? *Justify* your answer.
Yes; the incenter will allow the largest tent possible.

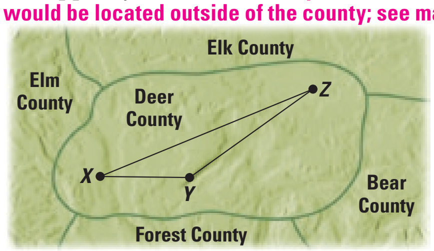
38. CHALLENGE You have seen that there is a point inside any triangle that is equidistant from the three sides of the triangle. Prove that if you extend the sides of the triangle to form lines, you can find three points outside the triangle, each of which is equidistant from those three lines.



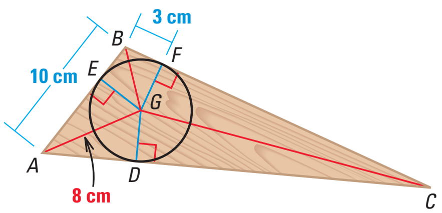
Sample answer: **Construct three circles exterior to the triangle, each one tangent to one side of the triangle and the other two lines. The centers of the circles are the three points.**

MIXED REVIEW *of Problem Solving*

1. **SHORT RESPONSE** A committee has decided to build a park in Deer County. The committee agreed that the park should be equidistant from the three largest cities in the county, which are labeled X, Y, and Z in the diagram. *Explain* why this may not be the best place to build the park. Use a sketch to support your answer. ***Sample answer:*** **The park would be located outside of the county; see margin for art.**



2. **EXTENDED RESPONSE** A woodworker is trying to cut as large a wheel as possible from a triangular scrap of wood. The wheel just touches each side of the triangle as shown below.



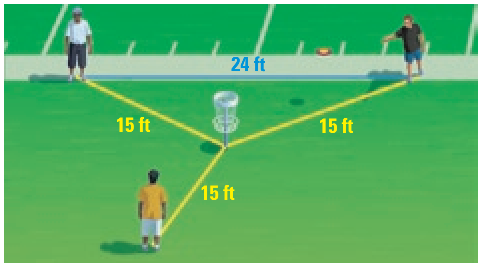
   a. Which point of concurrency is the woodworker using for the center of the circle? What type of special segment are $\overline{BG}$, $\overline{CG}$, and $\overline{AG}$? **incenter; angle bisectors**

   b. Which postulate or theorem can you use to prove that $\triangle BGF \cong \triangle BGE$? **HL**

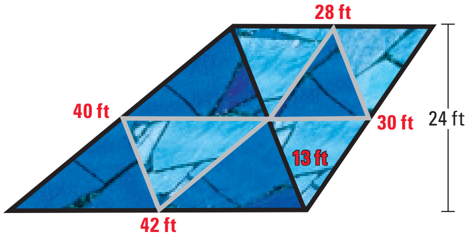
   c. Find the radius of the wheel to the nearest tenth of a centimeter. *Explain* your reasoning.
   3.9 cm; $(AE)^2 + (EG)^2 = (GA)^2$ or $7^2 + (EG)^2 = 8^2$

3. **SHORT RESPONSE** Graph $\triangle GHJ$ with vertices $G(2, 2)$, $H(6, 8)$, and $J(10, 4)$ and draw its midsegments. Each midsegment is contained in a line. Which of those lines has the greatest y-intercept? Write the equation of that line. *Justify* your answer. **$\overleftrightarrow{AC}$; $y = -x + 9$; the y-intercepts are -6, 4, and 9. The slope of the line with 9 as the y-intercept is -1 so the equation of that line is $y = -1x + 9$; see margin for art.**

4. **GRIDDED ANSWER** Three friends are practicing disc golf, in which a flying disk is thrown into a set of targets. Each player is 15 feet from the target. Two players are 24 feet from each other along one edge of the nearby football field. How far is the target from that edge of the football field? **9 ft**



5. **MULTI-STEP PROBLEM** An artist created a large floor mosaic consisting of eight triangular sections. The grey segments are the midsegments of the two black triangles.

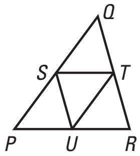


   a. The gray and black edging was created using special narrow tiles. What is the total length of all the edging used? **262 ft**

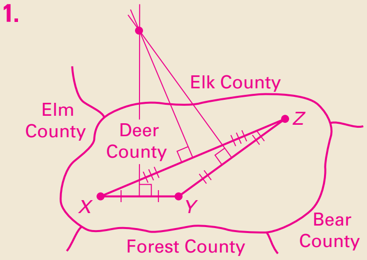
   b. What is the total area of the mosaic? **840 ft^2**

6. **OPEN-ENDED** If possible, draw a triangle whose incenter and circumcenter are the same point. *Describe* this triangle as specifically as possible. **Equilateral triangle; see margin for art.**

7. **SHORT RESPONSE** Points S, T, and U are the midpoints of the sides of $\triangle PQR$. Which angles are congruent to $\angle QST$? *Justify* your answer. **See margin.**



1.



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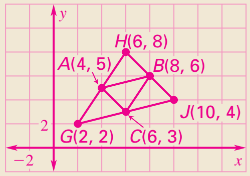
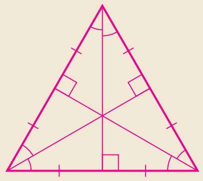


6.



7. $\angle QPR$, $\angle STU$, $\angle TUR$; $\overline{ST} \parallel \overline{PR}$ with $\overline{QP}$ a transversal, so $\angle QPR$ and $\angle QST$ are corresponding angles. $\overline{PQ} \parallel \overline{UT}$ with $\overline{ST}$ a transversal, so $\angle QST$ and $\angle STU$ are alternate interior angles. $\overline{ST} \parallel \overline{PR}$ with $\overline{TU}$ a transversal, so $\angle STU$ and $\angle TUR$ are alternate interior angles and $\angle QST \cong \angle TUR$ by the Transitive Property.

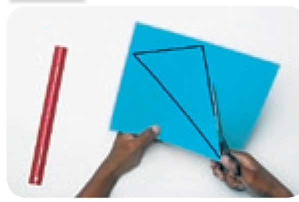
Intersecting Medians

MATERIALS • cardboard • straightedge • scissors • metric ruler

QUESTION What is the relationship between segments formed by the medians of a triangle?

EXPLORE 1 Find the balance point of a triangle

STEP 1



Cut out triangle Draw a triangle on a piece of cardboard. Then cut it out.

STEP 2

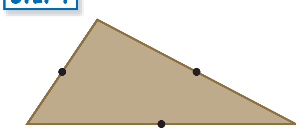
Balance the triangle Balance the triangle on the eraser end of a pencil.

STEP 3

Mark the balance point Mark the point on the triangle where it balanced on the pencil.

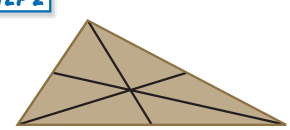
EXPLORE 2 Construct the medians of a triangle

STEP 1



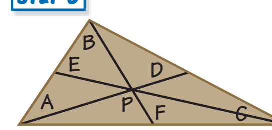
Find the midpoint Use a ruler to find the midpoint of each side of the triangle.

STEP 2



Draw medians Draw a segment, or *median*, from each midpoint to the vertex of the opposite angle.

STEP 3



Label points Label your triangle as shown. What do you notice about point *P* and the balance point in Explore 1?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Copy and complete the table. Measure in millimeters. **See margin.**

Length of segment from vertex to midpoint of opposite side	$AD = ?$	$BF = ?$	$CE = ?$
Length of segment from vertex to *P*	$AP = ?$	$BP = ?$	$CP = ?$
Length of segment from *P* to midpoint	$PD = ?$	$PF = ?$	$PE = ?$

2. How does the length of the segment from a vertex to *P* compare with the length of the segment from *P* to the midpoint of the opposite side? **It is twice as long.**

3. How does the length of the segment from a vertex to *P* compare with the length of the segment from the vertex to the midpoint of the opposite side? **It is $\frac{2}{3}$ the length.**

1. ***Sample:*** **Row 1: 27, 15, 36; Row 2: 18, 10, 24; Row 3: 9, 5, 12**

1 PLAN AND PREPARE

Explore the Concept

- Students will find the balance point of a triangle.
- This activity leads into the study of the centroid in this lesson, Example 1.

Materials

Each student or group of students will need:

- cardboard, straightedge
- scissors, metric ruler

Recommended Time

Work activity: 10 min

Discuss results: 5 min

Grouping

Students can work individually or in groups of two. In groups, students can alternate finding measurements.

2 TEACH

Tips for Success

Make sure students measure accurately so they can make valid conclusions.

Key Question

- What can you conclude about a line that contains a vertex and the balance point? **The line bisects the opposite side.**

Alternative Strategy

Show that the segment through the balance point, a vertex, and a point on the opposite side is a median.

Key Discovery

The medians of a triangle intersect in the balance point of a triangle.

3 ASSESS AND RETEACH

1. If P is the point of intersection of the medians of $\triangle ABC$ and $\overline{BF}$ is a median, how is BP related to PF? **$BP = 2 \cdot PF$**

5.4 Use Medians and Altitudes

Before You used perpendicular bisectors and angle bisectors of triangles.

Now You will use medians and altitudes of triangles.

Why? So you can find the balancing point of a triangle, as in Ex. 37.

Key Vocabulary
- median of a triangle
- centroid
- altitude of a triangle
- orthocenter

A triangle will balance at a particular point. This point is the intersection of the *medians* of the triangle.

A **median of a triangle** is a segment from a vertex to the midpoint of the opposite side. The three medians of a triangle are concurrent. The point of concurrency, called the **centroid**, is inside the triangle.

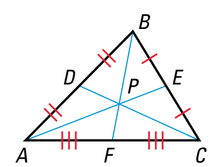
Three medians meet at the centroid.

THEOREM *For Your Notebook*

THEOREM 5.8 Concurrency of Medians of a Triangle

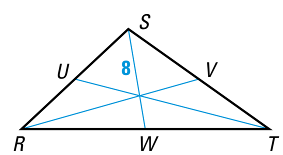
The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

The medians of $\triangle ABC$ meet at P and $AP = \frac{2}{3}AE$, $BP = \frac{2}{3}BF$, and $CP = \frac{2}{3}CD$.



EXAMPLE 1 Use the centroid of a triangle

In $\triangle RST$, Q is the centroid and $SQ = 8$. Find QW and SW.



Solution

$SQ = \frac{2}{3}SW$	Concurrency of Medians of a Triangle Theorem
$8 = \frac{2}{3}SW$	Substitute 8 for SQ.
$12 = SW$	Multiply each side by the reciprocal, $\frac{3}{2}$.

Then $QW = SW - SQ = 12 - 8 = 4$.

▶ So, $QW = 4$ and $SW = 12$.

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. For $A(-4, 8)$ and $B(5, 8)$, find the midpoint of $\overline{AB}$. $\left(\frac{1}{2}, 8\right)$
2. For $A(-3, 2)$ and $B(4, -1)$, find the length of $\overline{AB}$. $\sqrt{58}$
3. For $A(0, 4)$ and $C(18, 4)$, find the length of $\overline{AB}$, where B is a point $\frac{2}{3}$ the distance from A to C. 12

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 0.5 block with previous lesson
0.5 block with next lesson

- See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How do you find the centroid of a triangle? **Tell students they will learn how to answer this question by finding the point where the medians of a triangle intersect.**

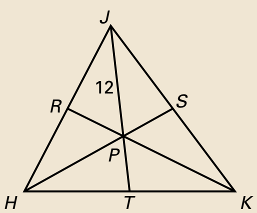
Motivating the Lesson

A microphone hangs on wires from two ceiling hooks. In the triangle formed by the hooks and the mike, what segment represents the distance from the mike to the ceiling? This lesson deals with that segment, the altitude, and other segments in triangles.

3 TEACH

Extra Example 1

In $\triangle HJK$, P is the centroid and $JP = 12$. Find PT and JT. **6; 18**



Key Questions to Ask for Example 1

- If $\triangle RST$ is scalene, does $SQ = RQ$? **no**
- Can the centroid and the incenter be the same point? **yes**

Extra Example 2

The vertices of $\triangle ABC$ are $A(1, 5)$, $B(5, 7)$, and $C(9, 3)$. Which ordered pair gives the coordinates of the centroid of $\triangle ABC$? **C**

Ⓐ (3, 6) Ⓑ (5, 4)
Ⓒ (5, 5) Ⓓ (7, 5)

Key Question to Ask for Example 2

- How would you find the coordinates of the centroid using the median from F? **The midpoint of $\overline{GH}$ is (5, 5) and the length of the segment from (5, 5) to $F(2, 5)$ is 3 units. The centroid is $\frac{2}{3}$ of that distance along a horizontal line from F, which is $P(4, 5)$.**

EXAMPLE 2 Standardized Test Practice

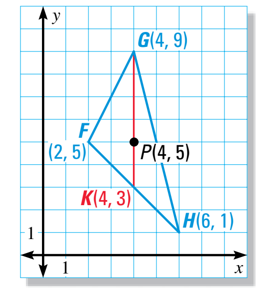
The vertices of $\triangle FGH$ are $F(2, 5)$, $G(4, 9)$, and $H(6, 1)$. Which ordered pair gives the coordinates of the centroid P of $\triangle FGH$?

Ⓐ (3, 5) Ⓑ (4, 5) Ⓒ (4, 7) Ⓓ (5, 3)

Solution

Sketch $\triangle FGH$. Then use the Midpoint Formula to find the midpoint K of $\overline{FH}$ and sketch median $\overline{GK}$.

$$K\left(\frac{2+6}{2}, \frac{5+1}{2}\right) = K(4, 3).$$



The centroid is two thirds of the distance from each vertex to the midpoint of the opposite side.

The distance from vertex $G(4, 9)$ to $K(4, 3)$ is $9 - 3 = 6$ units. So, the centroid is $\frac{2}{3}(6) = 4$ units down from G on $\overline{GK}$.

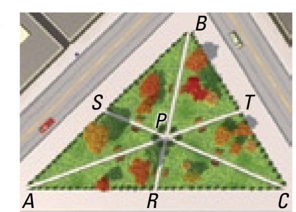
The coordinates of the centroid P are $(4, 9 - 4)$, or $(4, 5)$.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

CHECK ANSWERS
Median $\overline{GK}$ was used in Example 2 because it is easy to find distances on a vertical segment. It is a good idea to check by finding the centroid using a different median.

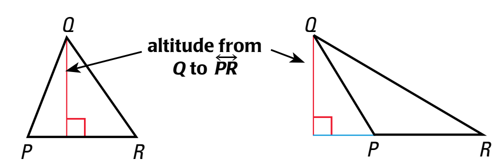
✓ GUIDED PRACTICE for Examples 1 and 2

There are three paths through a triangular park. Each path goes from the midpoint of one edge to the opposite corner. The paths meet at point P.



1. If $SC = 2100$ feet, find PS and PC. **700 ft, 1400 ft**
2. If $BT = 1000$ feet, find TC and BC. **1000 ft, 2000 ft**
3. If $PT = 800$ feet, find PA and TA. **1600 ft, 2400 ft**

ALTITUDES An **altitude of a triangle** is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.



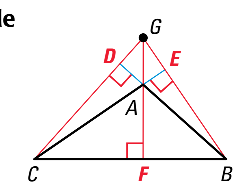
MEASURES OF TRIANGLES
In the area formula for a triangle, $A = \frac{1}{2}bh$, you can use the length of any side for the base b. The height h is the length of the altitude to that side from the opposite vertex.

THEOREM *For Your Notebook*

THEOREM 5.9 Concurrency of Altitudes of a Triangle

The lines containing the altitudes of a triangle are concurrent.

The lines containing $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ meet at G.



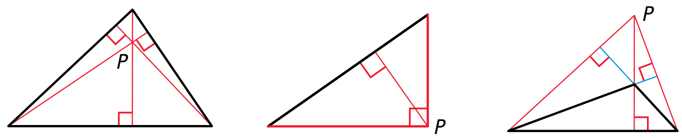
CONCURRENCY OF ALTITUDES The point at which the lines containing the three altitudes of a triangle intersect is called the **orthocenter** of the triangle.

EXAMPLE 3 Find the orthocenter

Find the orthocenter P in an acute, a right, and an obtuse triangle.

READ DIAGRAMS
The altitudes are shown in red. Notice that in the right triangle the legs are also altitudes. The altitudes of the obtuse triangle are extended to find the orthocenter.

Solution



Acute triangle	**Right triangle**	**Obtuse triangle**
***P* is inside triangle.**	***P* is on triangle.**	***P* is outside triangle.**

Animated Geometry at my.hrw.com

ISOSCELES TRIANGLES In an isosceles triangle, the perpendicular bisector, angle bisector, median, and altitude from the vertex angle to the base are all the same segment. In an equilateral triangle, this is true for the special segment from any vertex.

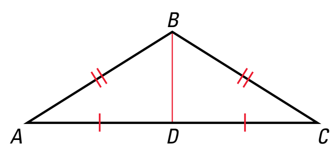
EXAMPLE 4 Prove a property of isosceles triangles

Prove that the median to the base of an isosceles triangle is an altitude.

Solution

GIVEN ▶ $\triangle ABC$ is isosceles, with base $\overline{AC}$.
$\overline{BD}$ is the median to base $\overline{AC}$.

PROVE ▶ $\overline{BD}$ is an altitude of $\triangle ABC$.



Proof Legs $\overline{AB}$ and $\overline{BC}$ of isosceles $\triangle ABC$ are congruent. $\overline{CD} \cong \overline{AD}$ because $\overline{BD}$ is the median to $\overline{AC}$. Also, $\overline{BD} \cong \overline{BD}$. Therefore, $\triangle ABD \cong \triangle CBD$ by the SSS Congruence Postulate.

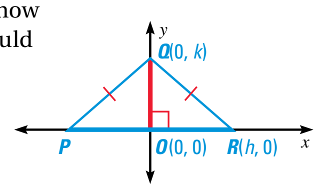
$\angle ADB \cong \angle CDB$ because corresponding parts of $\cong$ $\triangle$s are $\cong$. Also, $\angle ADB$ and $\angle CDB$ are a linear pair. $\overline{BD}$ and $\overline{AC}$ intersect to form a linear pair of congruent angles, so $\overline{BD} \perp \overline{AC}$ and $\overline{BD}$ is an altitude of $\triangle ABC$.

✓ **GUIDED PRACTICE** for Examples 3 and 4

4. Copy the triangle in Example 4 and find its orthocenter. **See margin.**

5. **WHAT IF?** In Example 4, suppose you wanted to show that median $\overline{BD}$ is also an angle bisector. How would your proof be different?

6. Triangle PQR is an isosceles triangle and segment $\overline{OQ}$ is an altitude. What else do you know about $\overline{OQ}$? What are the coordinates of P?

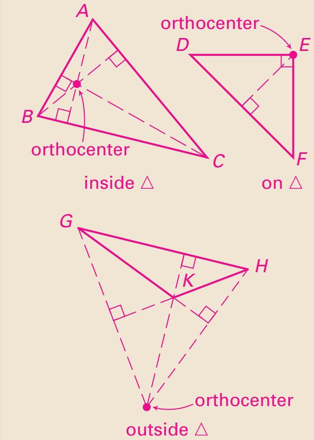


5. $\triangle ABD \cong \triangle CBD$ by SSS making $\angle ABD \cong \angle CBD$ which leads to $\overline{BD}$ being an angle bisector.

6. $\overline{OQ}$ is also a perpendicular bisector, angle bisector, and median; $(-h, 0)$.

Extra Example 3

Show that the orthocenter can be inside, on, or outside the triangle.



Animated Geometry my.hrw.com

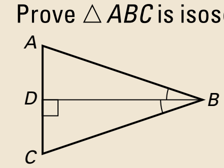
An **Animated Geometry** activity is available online for **Example 3**. This activity is also part of **Power Presentations**.

Extra Example 4

Prove that if an angle bisector of a triangle is also an altitude, then the triangle is isosceles.

Given $\triangle ABC$, with $\overline{BD}$ an angle bisector and altitude to $\overline{AC}$.

Prove $\triangle ABC$ is isosceles.



$\overline{BD}$ is an angle bisector and altitude, so $\angle ABD \cong \angle CBD$ and $\overline{BD} \perp \overline{AC}$. Then $\angle ADB$ and $\angle CDB$ are congruent right angles. Since $\overline{BD} \cong \overline{BD}$, $\triangle ABD \cong \triangle CBD$ by ASA. $\overline{AB} \cong \overline{CB}$ since they are corresponding parts of congruent triangles. So $\triangle ABC$ is isosceles.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you find the centroid of a triangle?

- **The medians of a triangle intersect at the centroid.**

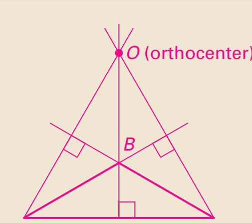
Find the midpoints of the sides and draw the medians. Locate the point where the medians intersect.

Differentiated Instruction

English Learners Students who are learning English may have difficulty distinguishing between the terms *bisector*, *median*, and *altitude* and between the names for the points where they intersect. On an index card, have students list the terms perpendicular bisector, angle bisector, median, and altitude, and draw a diagram to represent each as well as name the point where the lines intersect.

See also the *Differentiated Instruction Resources* for more strategies.

4.



4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–11, 25–27
Day 2:
Exs. 12–24, 28, 37–41

Average:
Day 1:
Exs. 1, 2, 4, 5, 7–11, 25–27, 33–35
Day 2:
Exs. 12–22 even, 23, 24, 28–32, 37–44

Advanced:
Day 1:
Exs. 1, 2, 5–11, 25–27, 33–35
Day 2:
Exs. 12, 14, 15, 21–24, 28–32, 36*, 39–45*

Block:
Exs. 1, 2, 4, 5, 7–11, 25–27, 33–35, 46–49, 53–55 (with previous lesson)
Exs. 12–22 even, 23, 24, 28–32, 37–44 (with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 8, 13, 18, 38
Average: 4, 10, 14, 20, 42
Advanced: 6, 11, 15, 22, 42

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

5.4 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 21, and 39
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 11, 12, 28, 40, and 44

SKILL PRACTICE

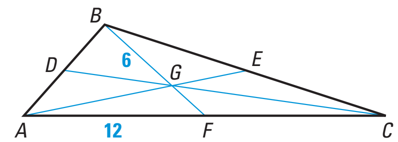
A circumcenter: when it is an acute triangle, when it is a right triangle, when it is an obtusetriangle; incenter: always, never, never; centroid: always, never, never; orthocenter: when it is an acute triangle, when it is a right triangle, when it is an obtuse triangle

1. **VOCABULARY** Name the four types of points of concurrency. When is each type inside the triangle? on the triangle? outside the triangle?

2. ★ **WRITING** *Compare* a perpendicular bisector and an altitude of a triangle. *Compare* a perpendicular bisector and a median of a triangle. **See margin.**

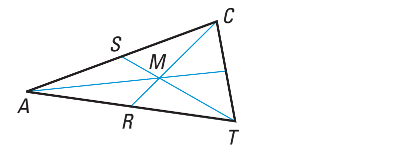
EXAMPLE 1 for Exs. 3–7

FINDING LENGTHS **G is the centroid of $\triangle ABC$, $BG = 6$, $AF = 12$, and $AE = 15$. Find the length of the segment.**



3. $\overline{FC}$ **12**
4. $\overline{BF}$ **9**
5. $\overline{AG}$ **10**
6. $\overline{GE}$ **5**

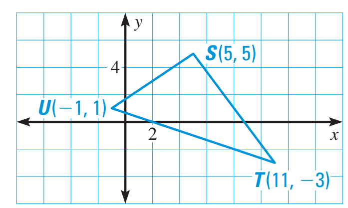
7. ★ **MULTIPLE CHOICE** In the diagram, M is the centroid of $\triangle ACT$, $CM = 36$, $MQ = 30$, and $TS = 56$. What is AM? **D**



Ⓐ 15
Ⓑ 30
Ⓒ 36
Ⓓ 60

EXAMPLE 2 for Exs. 8–11

8. **FINDING A CENTROID** Use the graph shown.



   a. Find the coordinates of P, the midpoint of $\overline{ST}$. Use the median $\overline{UP}$ to find the coordinates of the centroid Q. **(8, 1); (5, 1)**

   b. Find the coordinates of R, the midpoint of $\overline{TU}$. Verify that $SQ = \frac{2}{3}SR$. **(5, −1); $SQ = 4$ and $SR = 6$ therefore $SQ = \frac{2}{3}SR$.**

GRAPHING CENTROIDS **Find the coordinates of the centroid P of $\triangle ABC$.**

9. $A(-1, 2)$, $B(5, 6)$, $C(5, -2)$ **(3, 2)**

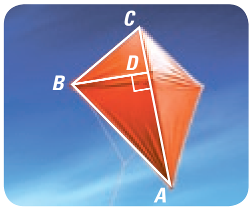
10. $A(0, 4)$, $B(3, 10)$, $C(6, -2)$ **(3, 4)**

11. ★ **OPEN-ENDED MATH** Draw a large right triangle and find its centroid. **See margin.**

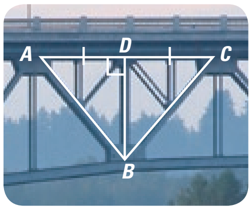
EXAMPLE 3 for Exs. 12–16

12. ★ **OPEN-ENDED MATH** Draw a large obtuse, scalene triangle and find its orthocenter. **See margin.**

IDENTIFYING SEGMENTS **Is $\overline{BD}$ a *perpendicular bisector* of $\triangle ABC$? Is $\overline{BD}$ a *median*? an *altitude*?**

13.


no; no; yes

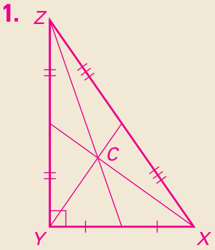
14.


yes; yes; yes

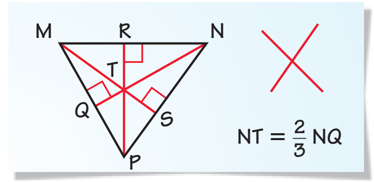
15.


no; yes; no

2. Both are perpendicular to a side of the triangle although the altitude contains the vertex opposite the side while a perpendicular bisector bisects the side but does not necessarily contain the opposite vertex; both bisect one side of a triangle although the perpendicular bisector does not necessarily contain the opposite vertex while the median is not necessarily perpendicular to the side but does contain the opposite vertex.

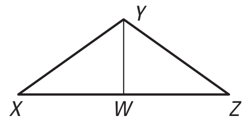
11.


16. **ERROR ANALYSIS** A student uses the fact that T is a point of concurrency to conclude that $NT = \frac{2}{3}NQ$. *Explain* what is wrong with this reasoning.
T is the orthocenter, but the centroid is needed to reach the conclusion.



EXAMPLE 4
for Exs. 17–22

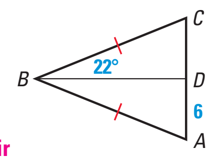
REASONING **Use the diagram shown and the given information to decide whether $\overline{YW}$ is a *perpendicular bisector*, an *angle bisector*, a *median*, or an *altitude* of $\triangle XYZ$. There may be more than one right answer.** 20–22. See margin.



17. $\overline{YW} \perp \overline{XZ}$ altitude
18. $\angle XYW \cong \angle ZYW$ angle bisector
19. $\overline{XW} \cong \overline{ZW}$ median
20. $\overline{YW} \perp \overline{XZ}$ and $\overline{XW} \cong \overline{ZW}$
21. $\triangle XYW \cong \triangle ZYW$
22. $\overline{YW} \perp \overline{XZ}$ and $\overline{XY} \cong \overline{ZY}$

B

ISOSCELES TRIANGLES **Find the measurements. *Explain* your reasoning.**



23. Given that $\overline{DB} \perp \overline{AC}$, find DC and $m\angle ABD$.
6, 22°; $\triangle ABD \cong \triangle CBD$ by HL, use Corr. parts of $\cong$ $\triangle$s are $\cong$.
24. Given that $AD = DC$, find $m\angle ADB$ and $m\angle ABD$.
90°, 22°; $\triangle ABD \cong \triangle CBD$ by SSS, use definition of a linear pair and Corr. parts of $\cong$ $\triangle$s are $\cong$.

RELATING LENGTHS **Copy and complete the statement for $\triangle DEF$ with medians $\overline{DH}$, $\overline{EJ}$, and $\overline{FG}$, and centroid K.**

25. $EJ = \underline{\ ?\ }\ KJ$ 3
26. $DK = \underline{\ ?\ }\ KH$ 2
27. $FG = \underline{\ ?\ }\ KF$ $\frac{3}{2}$

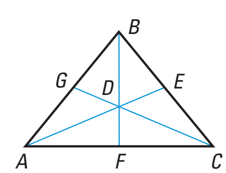
28. ★ **SHORT RESPONSE** Any isosceles triangle can be placed in the coordinate plane with its base on the x-axis and the opposite vertex on the y-axis as in Guided Practice Exercise 6 in this lesson. *Explain* why. If the base angles of the isosceles triangle are placed at $(-a, 0)$ and $(0, a)$, the vertex angle will be on the y-axis.

CONSTRUCTION **Verify the Concurrency of Altitudes of a Triangle by drawing a triangle of the given type and constructing its altitudes.** 29–31. See margin.

29. Equilateral triangle
30. Right scalene triangle
31. Obtuse isosceles triangle

32. **VERIFYING THEOREM 5.8** Use Example 2 in this lesson. Verify that Theorem 5.8, the Concurrency of Medians of a Triangle, holds for the median from vertex F and for the median from vertex H. See margin.

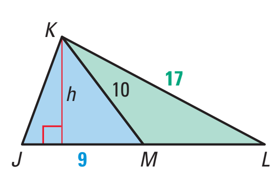
xy **ALGEBRA** **Point D is the centroid of $\triangle ABC$. Use the given information to find the value of x.**



33. $BD = 4x + 5$ and $BF = 9x$ $\frac{5}{2}$
34. $GD = 2x - 8$ and $GC = 3x + 3$ 9
35. $AD = 5x$ and $DE = 3x - 2$ 4

C

36. **CHALLENGE** $\overline{KM}$ is a median of $\triangle JKL$. Find the areas of $\triangle JKM$ and $\triangle LKM$. Compare the areas. Do you think that the two areas will always compare in this way, regardless of the shape of the triangle? *Explain*.
$\frac{9\sqrt{19}}{2}$, $\frac{9\sqrt{19}}{2}$; yes; the height and base of both triangles will always be the same.



20. perpendicular bisector, angle bisector, median, altitude

21. perpendicular bisector, angle bisector, median, altitude

22. perpendicular bisector, angle bisector, median, altitude

Vocabulary

Exercises 17–22 Have students review the definitions of the terms in italic before they do these exercises.

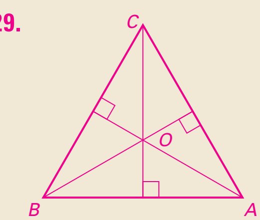
Avoiding Common Errors

Exercises 25–27 Some students may not be able to visualize the position of a centroid in a triangle. Encourage students to draw and label the centroid for each triangle.

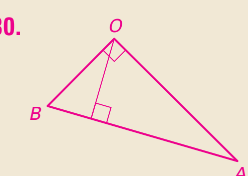
Teaching Strategy

Exercise 36 Because *M* is a mid-point, the length of the bases of the two triangles are equal. If two triangles have the same base and same altitude to that base, they have the same area. Geometry software can be used to help students see this relationship for this exercise.

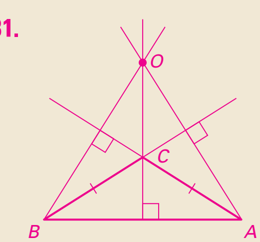
29.



30.

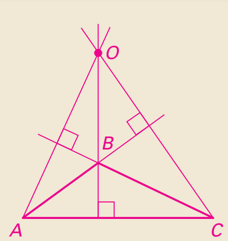


31.



32. *Sample answer:* The midpoint of $\overline{FG}$ is $L(3, 7)$, so the equation of the median from $H(6, 1)$ to $L(3, 7)$ is $y = -2x + 13$. $P(4, 5)$ lies on this median. The midpoint of $\overline{GH}$ is $J(5, 5)$, so the equation of the median from $F(2, 5)$ to $J(5, 5)$ is $y = 5$. $P(4, 5)$ lies on this median, so all three medians intersect at the centroid.

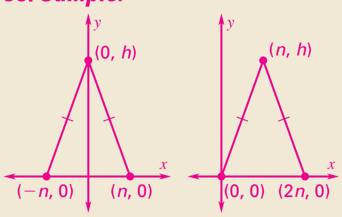
12.



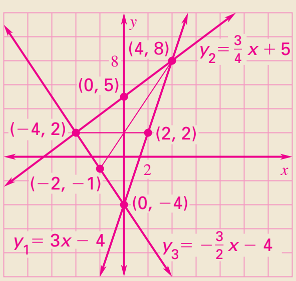
Teaching Strategy

Exercise 41 Students can start by graphing the lines and observing where they intersect. After students solve systems of pairs of the equations to find the exact coordinates of the vertices, they can use the midpoint formula to find equations of the three medians. Finally, students can solve a system for any two of the medians to find the centroid.

38. *Sample:*



41.



42a. Statements (Reasons)

1. $\triangle ABC$ is equilateral, $\overline{BD}$ is an altitude of $\triangle ABC$. (Given)
2. $\overline{AB} \cong \overline{BC}$ (Definition of equilateral triangle)
3. $\overline{BD} \cong \overline{BD}$ (Reflexive Property of Segment Congruence)
4. $\angle ADB$ and $\angle CDB$ are right angles. (Definition of altitude)
5. $\triangle ABD$ and $\triangle CBD$ are right triangles. (Definition of right triangle)
6. $\triangle ABD \cong \triangle CBD$ (HL)
7. $\overline{AD} \cong \overline{CD}$ (Corr. parts of $\cong$ $\triangle$s are $\cong$.)
8. $AD = CD$ (Definition of congruent segments)
9. $\overline{BD}$ is a perpendicular bisector of $\overline{AC}$. (Definition of perpendicular bisector)

PROBLEM SOLVING

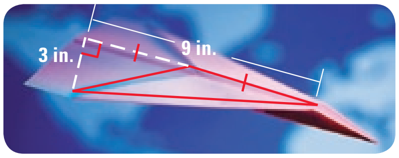
A

37. MOBILES To complete the mobile, you need to balance the red triangle on the tip of a metal rod. Copy the triangle and decide if you should place the rod at *A* or *B*. *Explain.*

***B*; it is the centroid of the triangle.**

38. DEVELOPING PROOF Show two different ways that you can place an isosceles triangle with base $2n$ and height h on the coordinate plane. Label the coordinates for each vertex. **See margin.**

39. PAPER AIRPLANE Find the area of the triangular part of the paper airplane wing that is outlined in red. Which special segment of the triangle did you use?



6.75 in.^{2}; altitude

B

40. ★ SHORT RESPONSE In what type(s) of triangle can a vertex of the triangle be one of the points of concurrency of the triangle? *Explain.*

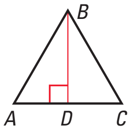
Right; the orthocenter is on the right angle.

41. COORDINATE GEOMETRY Graph the lines on the same coordinate plane and find the centroid of the triangle formed by their intersections. **(0, 2); see margin for art.**

$y_1 = 3x - 4$ $y_2 = \frac{3}{4}x + 5$ $y_3 = -\frac{3}{2}x - 4$

EXAMPLE 4 for Ex. 42

42. PROOF Write proofs using different methods. **See margin.**



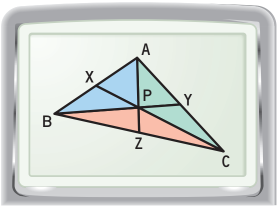
GIVEN ▶ $\triangle ABC$ is equilateral.
$\overline{BD}$ is an altitude of $\triangle ABC$.

PROVE ▶ $\overline{BD}$ is also a perpendicular bisector of $\overline{AC}$.

a. Write a proof using congruent triangles.

b. Write a proof using the Perpendicular Postulate.

43. TECHNOLOGY Use geometry drawing software.



a. Construct a triangle and its medians. Measure the areas of the blue, green, and red triangles. **Check students' work.**

b. What do you notice about the triangles? **Their areas are the same.**

c. If a triangle is of uniform thickness, what can you conclude about the weight of the three interior triangles? How does this support the idea that a triangle will balance on its centroid?

They weigh the same; it means the weight of $\triangle ABC$ is evenly distributed around its centroid.

44. ★ EXTENDED RESPONSE Use $P(0, 0)$, $Q(8, 12)$, and $R(14, 0)$.

a. What is the slope of the altitude from R to $\overline{PQ}$? $-\frac{2}{3}$

b. Write an equation for each altitude of $\triangle PQR$. Find the orthocenter by finding the ordered pair that is a solution of the three equations. **$y = \frac{1}{2}x$, $x = 8$, $y = -\frac{2}{3}x + \frac{28}{3}$; (8, 4)**

c. How would your steps change if you were finding the circumcenter?

Find the equation of each perpendicular bisector of each side and solve the system.

○ = See **WORKED-OUT SOLUTIONS** in Student Resources   ★ = **STANDARDIZED TEST PRACTICE**

42b. Statements (Reasons)

1. $\triangle ABC$ is equilateral. (Given)
2. $BA = BC$ (Definition of equilateral triangle)
3. $\overline{BD} \perp \overline{AC}$ (Definition of altitude)
4. B is on the perpendicular bisector of $\overline{AC}$. (Converse of the Perpendicular Bisector Theorem)
5. Only one line exists through B perpendicular to $\overline{AC}$, so the perpendicular bisector and the altitude are contained in the same line. (Perpendicular Postulate)

C **45.** **CHALLENGE** Prove the results in parts (a) – (c). **See margin.**

GIVEN ▶ $\overline{LP}$ and $\overline{MQ}$ are medians of scalene $\triangle LMN$. Point R is on $\overrightarrow{LP}$ such that $\overline{LP} \cong \overline{PR}$. Point S is on $\overrightarrow{MQ}$ such that $\overline{MQ} \cong \overline{QS}$.

PROVE ▶ **a.** $\overline{NS} \cong \overline{NR}$

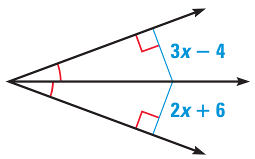
b. $\overline{NS}$ and $\overline{NR}$ are both parallel to $\overline{LM}$.

c. R, N, and S are collinear.

QUIZ

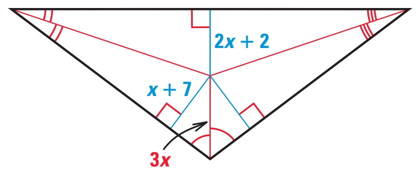
Find the value of x. Identify the theorem used to find the answer.

1.



10; Angle Bisector Theorem

2.



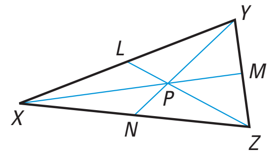
5; Concurrency of Angle Bisectors of a Triangle

In the figure, P is the centroid of $\triangle XYZ$, $YP = 12$, $LX = 15$, and $LZ = 18$.

3. Find the length of $\overline{LY}$. **15**

4. Find the length of $\overline{YN}$. **18**

5. Find the length of $\overline{LP}$. **6**

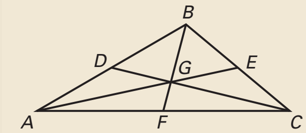


5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

For Exercises 1–3, use the diagram below. G is the centroid of $\triangle ABC$.

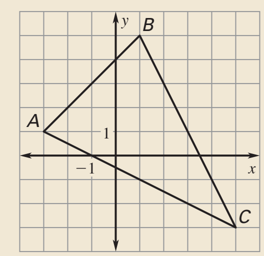


1. If $BG = 9$, find BF. **13.5**

2. If $BD = 12$, find AD. **12**

3. If $CD = 27$, find GC. **18**

4. Find the centroid of $\triangle ABC$. **(1, 1)**



5. Which type of triangle has its orthocenter on the triangle? **a right triangle**

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

45a–c. See Additional Answers.

1 PLAN AND PREPARE

Learn the Method

- Students will draw the perpendicular bisectors, medians, and altitudes of a triangle.
- Students can use this activity to verify the concurrency of each type of segment.

Keystroke Help

Keystrokes for several models of calculators are available in blackline format in the *Chapter Resource Book*.

2 TEACH

Tips for Success

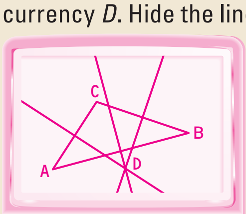
Have students experiment with different types of triangles. Ask them to drag the vertices to see that points *D*, *E*, and *F* remain collinear.

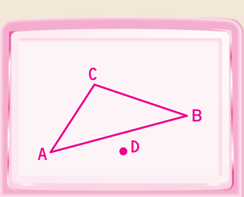
Alternative Strategy

You may want to do this activity as a demonstration. Have students write conclusions in their notebooks and share them with the class.

Extra Example 1

Draw the perpendicular bisectors of a triangle. Label the point of concurrency *D*. Hide the lines.





Investigate Points of Concurrency

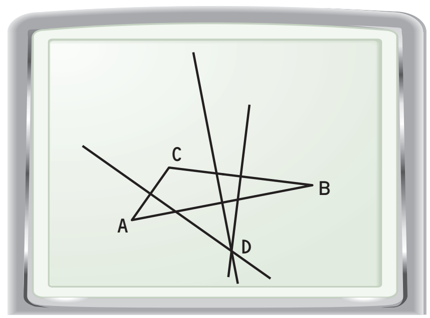
MATERIALS • graphing calculator or computer

QUESTION **How are the points of concurrency in a triangle related?**

You can use geometry drawing software to investigate concurrency.

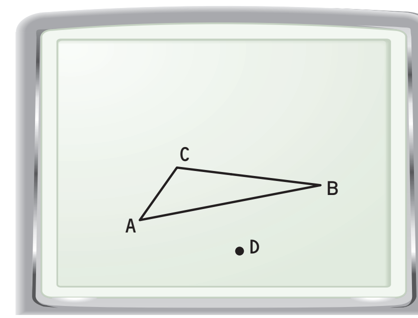
EXAMPLE 1 **Draw the perpendicular bisectors of a triangle**

STEP 1



Draw perpendicular bisectors Draw a line perpendicular to each side of a $\triangle ABC$ at the midpoint. Label the point of concurrency *D*.

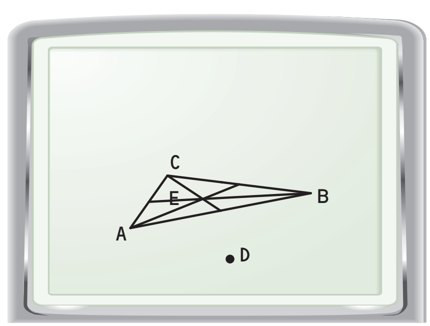
STEP 2



Hide the lines Use the *HIDE* feature to hide the perpendicular bisectors. Save as "EXAMPLE1."

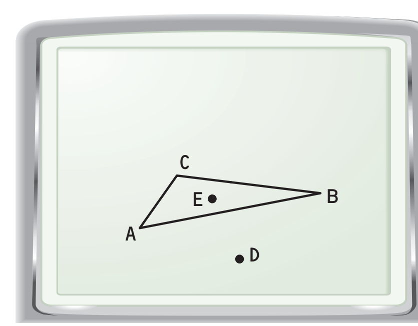
EXAMPLE 2 **Draw the medians of the triangle**

STEP 1



Draw medians Start with the figure you saved as "EXAMPLE1." Draw the medians of $\triangle ABC$. Label the point of concurrency *E*.

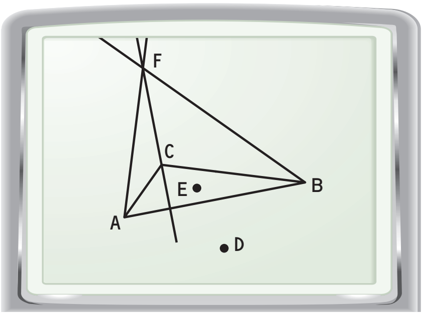
STEP 2



Hide the lines Use the *HIDE* feature to hide the medians. Save as "EXAMPLE2."

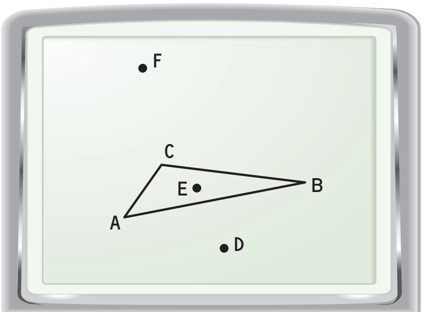
EXAMPLE 3 Draw the altitudes of the triangle

STEP 1



Draw altitudes Start with the figure you saved as "EXAMPLE2." Draw the altitudes of △*ABC*. Label the point of concurrency *F*.

STEP 2



Hide the lines Use the *HIDE* feature to hide the altitudes. Save as "EXAMPLE3."

PRACTICE

1. Try to draw a line through points *D*, *E*, and *F*. Are the points collinear? **yes**
2. Try dragging point *A*. Do points *D*, *E*, and *F* remain collinear? **yes**

In Exercises 3–5, use the triangle you saved as "EXAMPLE3."

3. Draw the angle bisectors. Label the point of concurrency as point *G*. **See margin.**
4. How does point *G* relate to points *D*, *E*, and *F*? **It is not collinear to all three points.**
5. Try dragging point *A*. What do you notice about points *D*, *E*, *F*, and *G*? ***D*, *E*, and *F* remain collinear; *G* is not collinear.**

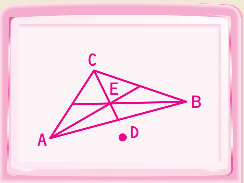
DRAW CONCLUSIONS

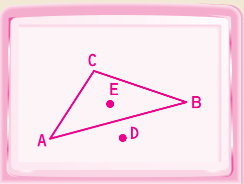
In 1765, Leonhard Euler (pronounced "oi′-ler") proved that the circumcenter, the centroid, and the orthocenter are all collinear. The line containing these three points is called *Euler's line*. Save the triangle from Exercise 5 as "EULER" and use that for Exercises 6–8.

6. Try moving the triangle's vertices. Can you verify that the same three points lie on Euler's line whatever the shape of the triangle? *Explain.* **6. Yes. *Sample answer:* After changing the shape of the triangle identify the circumcenter, centroid, and orthocenter to see that they remain collinear.**
7. Notice that some of the four points can be outside of the triangle. Which points lie outside the triangle? Why? What happens when you change the shape of the triangle? Are there any points that never lie outside the triangle? Why? **See margin.**
8. Draw the three midsegments of the triangle. Which, if any, of the points seem contained in the triangle formed by the midsegments? Do those points stay there when the shape of the large triangle is changed? **Centroid, incenter; yes; see margin for art.**

Extra Example 2

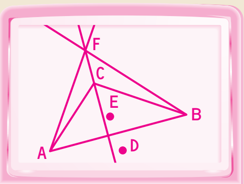
Draw the medians of the triangle. Label the point of concurrency *E*. Hide the lines.

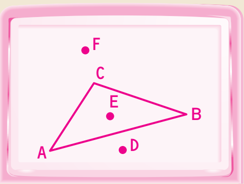




Extra Example 3

Draw the altitudes of the triangle. Label the point of concurrency *F*. Hide the lines.

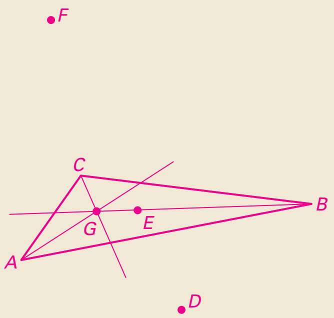




3 ASSESS AND RETEACH

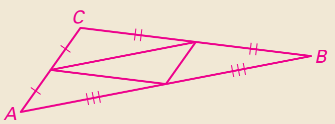
1. Describe the position of the points of the Euler line for each type of triangle.
   a. acute isosceles triangle **the same as the altitude to the base**
   b. isosceles right triangle **the same as the altitude to the hypotenuse**

3.



7. *Sample answer:* The circumcenter or orthocenter; the triangle is obtuse; if the triangle is acute, the circumcenter and orthocenter are inside. If the triangle is a right triangle, the circumcenter and orthocenter are on it. If the triangle is obtuse, the circumcenter and orthocenter are outside; the incenter and centroid; the medians and angle bisectors will always intersect in the interior of the triangle.

8.



1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. Solve $3x + 8 < 29$. $x < 7$
2. Solve $15 > -2x - 9$. $x > -12$
3. Solve $2x - 2 < (3x - 4) + (x - 8)$. $x > 5$
4. Find all integer solutions for $4 < x < 11$. 5, 6, 7, 8, 9, 10
5. A triangle has angle measures 82° and 34°. Find the measure of the third angle. 64°

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day
Average: 1 day
Advanced: 1 day
Block: 0.5 block with previous lesson

• See *Teaching Guide/Lesson Plan*.

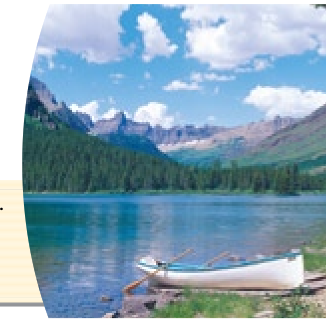
2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How do you find the possible lengths of the third side of a triangle if you know the lengths of two sides? **Tell students they will learn how to answer this question by applying the Triangle Inequality Theorem.**

5.5 Use Inequalities in a Triangle



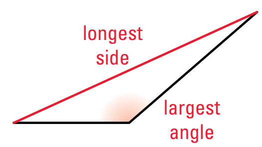
Before	You found what combinations of angles are possible in a triangle.
Now	You will find possible side lengths of a triangle.
Why?	So you can find possible distances, as in Ex. 39.

Key Vocabulary
- **side opposite**
- **inequality**

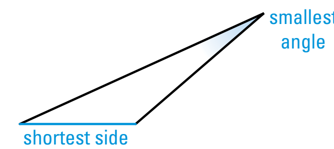
EXAMPLE 1 Relate side length and angle measure

Draw an obtuse scalene triangle. Find the largest angle and longest side and mark them in red. Find the smallest angle and shortest side and mark them in blue. What do you notice?

Solution



The longest side and largest angle are opposite each other.



The shortest side and smallest angle are opposite each other.

The relationships in Example 1 are true for all triangles as stated in the two theorems below. These relationships can help you to decide whether a particular arrangement of side lengths and angle measures in a triangle may be possible.

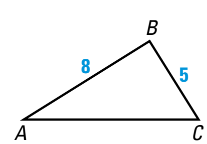
AVOID ERRORS

Be careful not to confuse the symbol ∠ meaning *angle* with the symbol < meaning *is less than.* Notice that the bottom edge of the angle symbol is horizontal.

THEOREMS — *For Your Notebook*

THEOREM 5.10

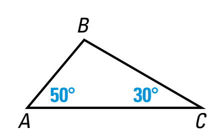
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



$AB > BC$, so $m\angle C > m\angle A$.

THEOREM 5.11

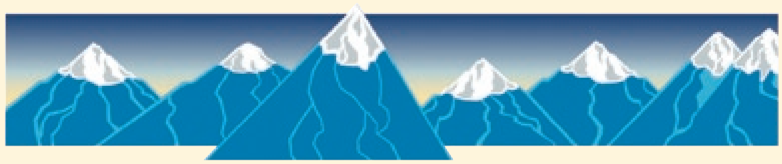
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.



$m\angle A > m\angle C$, so $BC > AB$.

EXAMPLE 2 Standardized Test Practice

STAGE PROP You are constructing a stage prop that shows a large triangular mountain. The bottom edge of the mountain is about 27 feet long, the left slope is about 24 feet long, and the right slope is about 20 feet long. You are told that one of the angles is about 46° and one is about 59°. What is the angle measure of the peak of the mountain?

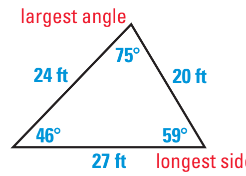


Ⓐ 46° Ⓑ 59° Ⓒ 75° Ⓓ 85°

ELIMINATE CHOICES
You can eliminate choice D because a triangle with a 46° angle and a 59° angle cannot have an 85° angle. The sum of the three angles in a triangle must be 180°, but the sum of 46, 59, and 85 is 190, not 180.

Solution

Draw a diagram and label the side lengths. The peak angle is opposite the longest side so, by Theorem 5.10, the peak angle is the largest angle.

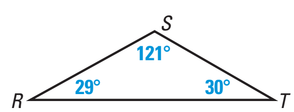


The angle measures sum to 180°, so the third angle measure is $180° - (46° + 59°) = 75°$. You can now label the angle measures in your diagram.

▶ The greatest angle measure is 75°, so the correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 1 and 2

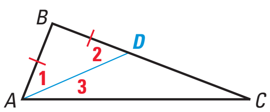
1. List the sides of $\triangle RST$ in order from shortest to longest. ***$\overline{ST}$, $\overline{RS}$, $\overline{TR}$***



2. Another stage prop is a right triangle with sides that are 6, 8, and 10 feet long and angles of 90°, about 37°, and about 53°. Sketch and label a diagram with the shortest side on the bottom and the right angle at the left. **See margin.**

PROOF Theorem 5.10

GIVEN ▶ $BC > AB$
PROVE ▶ $m\angle BAC > m\angle C$



Locate a point D on $\overline{BC}$ such that $DB = BA$. Then draw $\overline{AD}$. In the isosceles triangle $\triangle ABD$, $\angle 1 \cong \angle 2$.

Because $m\angle BAC = m\angle 1 + m\angle 3$, it follows that $m\angle BAC > m\angle 1$. Substituting $m\angle 2$ for $m\angle 1$ produces $m\angle BAC > m\angle 2$.

By the Exterior Angle Theorem, $m\angle 2 = m\angle 3 + m\angle C$, so it follows that $m\angle 2 > m\angle C$ (see Exercise 27 in this lesson). Finally, because $m\angle BAC > m\angle 2$ and $m\angle 2 > m\angle C$, you can conclude that $m\angle BAC > m\angle C$.

Motivating the Lesson

Give each student three straws of various lengths, some of which form triangles and some of which do not. Have students investigate when three straws can form a triangle, and when then cannot. Tell students that they will describe and examine this property in this lesson.

3 TEACH

Extra Example 1

Draw an acute scalene triangle. Find and label the largest angle and the longest side. Find and label the smallest angle and shortest side. What do you notice? **The longest side and largest angle are opposite each other. The shortest side and smallest angle are opposite each other.**

Extra Example 2

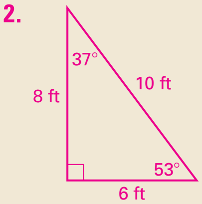
Three wooden beams will be nailed together to form a brace for a wall. The bottom edge of the brace is about 8 feet, and the sides are about 12 feet and 14 feet. One of the angles measures about 86° and the other measures about 35°. What is the angle measure opposite the largest side of the brace? **C**

Ⓐ 35° Ⓑ 59°
Ⓒ 86° Ⓓ 96°

Key Question to Ask for Example 2

- How do you know that the greatest angle is opposite the 27 foot edge? **The 27 ft edge is the longest side.**

2.



Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for the **Triangle Inequality**. This activity is also part of **Power Presentations**.

Extra Example 3

A triangle has one side of length 11 and another of length 6. Describe the possible lengths of the third side.
greater than 5 and less than 17

Key Questions to Ask for Example 3

- Can the third side have a length of 21? **no**
- Can the third side have a length of 5.5? **yes**

Teaching Strategy

It is important that students know how to interpret the compound inequality $4 < x < 20$ in the solution of Example 3. Emphasize that it is a shorter way to write the two inequalities $x > 4$ and $x < 20$.

Closing the Lesson

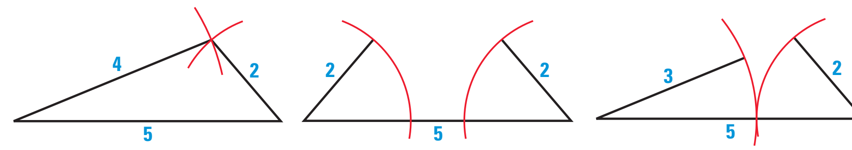
Have students summarize the major points of the lesson and answer the Essential Question: How do you find the possible lengths of the third side of a triangle if you know the lengths of two sides?

- **In a triangle, the largest angle is opposite the longest side and the smallest angle is opposite the shortest side.**
- **The sum of the lengths of any two sides of a triangle is greater than the length of the third side.**

The length of the third side can be any value greater than the difference of the two lengths and less than the sum of the two lengths.

THE TRIANGLE INEQUALITY Not every group of three segments can be used to form a triangle. The lengths of the segments must fit a certain relationship.

For example, three attempted triangle constructions for sides with given lengths are shown below. Only the first set of side lengths forms a triangle.



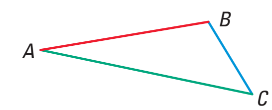
If you start with the longest side and attach the other two sides at its endpoints, you can see that the other two sides are not long enough to form a triangle in the second and third figures. This leads to the *Triangle Inequality Theorem.*

Animated Geometry at my.hrw.com

THEOREM *For Your Notebook*

THEOREM 5.12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



$AB + BC > AC$ $\quad$ $AC + BC > AB$ $\quad$ $AB + AC > BC$

EXAMPLE 3 Find possible side lengths

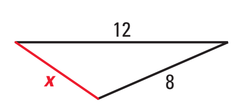
ALGEBRA **A triangle has one side of length 12 and another of length 8. Describe the possible lengths of the third side.**

Solution

Let x represent the length of the third side. Draw diagrams to help visualize the small and large values of x. Then use the Triangle Inequality Theorem to write and solve inequalities.

USE SYMBOLS
You can combine the two inequalities, $x > 4$ and $x < 20$, to write the compound inequality $4 < x < 20$. This can be read as *x is between 4 and 20.*

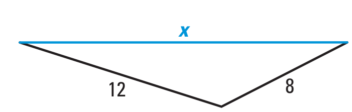
Small values of *x*



$x + 8 > 12$

$x > 4$

Large values of *x*



$8 + 12 > x$

$20 > x$, or $x < 20$

▶ The length of the third side must be greater than 4 and less than 20.

✓ GUIDED PRACTICE for Example 3

3. A triangle has one side of 11 inches and another of 15 inches. *Describe* the possible lengths of the third side. **$4 < x < 26$**

Differentiated Instruction

Below Level Give students a number of straws and ask them to cut them into various lengths. Then have them measure each length. Ask them to make a table listing the measures of many possible combinations of the three lengths. Then have them manipulate the straws to see if they can form a triangle. Highlight sets of three measurements that do not form a triangle.

See also the *Differentiated Instruction Resources* for more strategies.

5.5 EXERCISES

HOMEWORK KEY

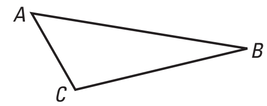
◯ = See **WORKED-OUT SOLUTIONS** Exs. 9, 17, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 20, 30, 39, and 45

SKILL PRACTICE

A

1. **VOCABULARY** Use the diagram at the right. For each angle, name the side that is *opposite* that angle. $\angle A$, $\overline{BC}$; $\angle B$, $\overline{CA}$; $\angle C$, $\overline{AB}$



2. ★ **WRITING** How can you tell from the angle measures of a triangle which side of the triangle is the longest? the shortest?
   It is opposite the largest angle; it is opposite the smallest angle.

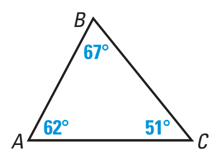
EXAMPLE 1 for Exs. 3–5

MEASURING **Use a ruler and protractor to draw the given type of triangle. Mark the largest angle and longest side in red and the smallest angle and shortest side in blue. What do you notice?** 3–5. See margin.

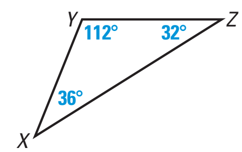
3. Acute scalene
4. Right scalene
5. Obtuse isosceles

EXAMPLE 2 for Exs. 6–15

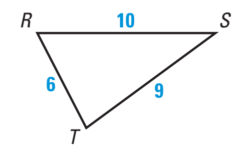
WRITING MEASUREMENTS IN ORDER **List the sides and the angles in order from smallest to largest.**

6.


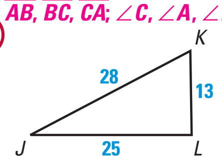
$\overline{AB}$, $\overline{BC}$, $\overline{CA}$; $\angle C$, $\angle A$, $\angle B$

7.


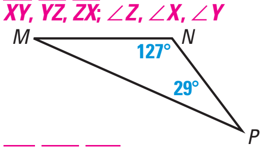
$\overline{XY}$, $\overline{YZ}$, $\overline{ZX}$; $\angle Z$, $\angle X$, $\angle Y$

8.


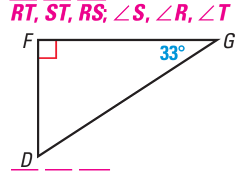
$\overline{RT}$, $\overline{ST}$, $\overline{RS}$; $\angle S$, $\angle R$, $\angle T$

9.


$\overline{KL}$, $\overline{JL}$, $\overline{JK}$; $\angle J$, $\angle K$, $\angle L$

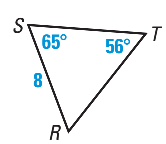
10.


$\overline{NP}$, $\overline{MN}$, $\overline{PM}$; $\angle M$, $\angle P$, $\angle N$

11.


$\overline{DF}$, $\overline{FG}$, $\overline{GD}$; $\angle G$, $\angle D$, $\angle F$

12. ★ **MULTIPLE CHOICE** In $\triangle RST$, which is a possible side length for ST? C



Ⓐ 7 Ⓑ 8

Ⓒ 9 Ⓓ Cannot be determined

DRAWING TRIANGLES **Sketch and label the triangle described.** 13–15. See margin.

13. Side lengths: about 3 m, 7 m, and 9 m, with longest side on the bottom
    Angle measures: 16°, 41°, and 123°, with smallest angle at the left

14. Side lengths: 37 ft, 35 ft, and 12 ft, with shortest side at the right
    Angle measures: about 71°, about 19°, and 90°, with right angle at the top

15. Side lengths: 11 in., 13 in., and 14 in., with middle-length side at the left
    Two angle measures: about 48° and 71°, with largest angle at the top

EXAMPLE 3 for Exs. 16–26

IDENTIFYING POSSIBLE TRIANGLES **Is it possible to construct a triangle with the given side lengths? If not, *explain* why not.**

16. 6, 7, 11 yes
17. 3, 6, 9 No; 3 + 6 is not greater than 9.
18. 28, 34, 39 yes
19. 35, 120, 125 yes

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–9, 12–17, 20–28, 37–42

Average:
Day 1:
Exs. 1–5, 7–9, 12–15, 17, 18, 20–26 even, 27–34, 38–45

Advanced:
Day 1:
Exs. 1, 2, 5, 10–12, 15, 18–20, 25–36*, 39–48*

Block:
Exs. 1–5, 7–9, 12–15, 17, 18, 20–26 even, 27–34, 38–45 (with previous lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

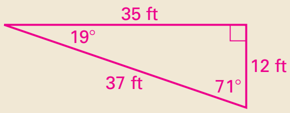
Basic: 3, 8, 16, 21, 39
Average: 4, 8, 22, 38, 39
Advanced: 5, 15, 26, 39, 40

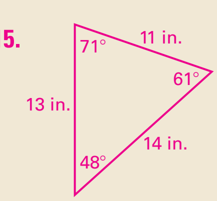
Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

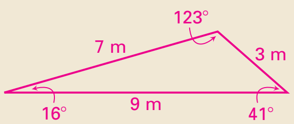
Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

14.


15.


3. *Sample answer:* The longest side is opposite the largest angle. The shortest side is opposite the smallest angle.
4. *Sample answer:* The largest angle is the right angle, and the longest side is the hypotenuse, opposite the right angle. The shortest side is opposite the smaller acute angle.
5. *Sample answer:* The longest side is opposite the obtuse angle, and the two angles with the same measure are opposite the sides with the same length.

13.


Study Strategy

Exercises 6–11 Some students may find it helpful to list all the angles and sides before doing these exercises. Then they can list the angles in order and write the side lengths opposite those angles.

Avoiding Common Errors

Exercises 13–15 Student should not attempt to make drawings "to scale" for these exercises. Have them start with a rough sketch and then refine it using all the conditions listed.

Teaching Strategy

Exercises 16–19 Ask students who are having difficulty with these problems to cut 3 strips of paper for each triangle with centimeter measures matching the side lengths. They can use their paper strips to try and form triangles.

Exercises 21–26 Ask students who are having difficulty with these problems to cut 2 strips of paper for each triangle with centimeter measures matching the side lengths. Then they can manipulate the sides to find the maximum and minimum lengths for the third side.

27. $\angle A$ and $\angle B$ are the nonadjacent interior angles to $\angle 1$ thus by the Exterior Angle Theorem $m\angle 1 = m\angle A + m\angle B$, which guarantees $m\angle 1 > m\angle A$ and $m\angle 1 > m\angle B$.

28. The diagram indicates that exterior angle of the triangle has the same measure as one of the nonadjacent interior angles, which cannot be.

35. $\angle WXY$, $\angle Z$, $\angle ZXY$, $\angle WYX$ and $\angle ZYX$, $\angle W$; $\angle ZYX$ is the largest angle in $\triangle ZYX$ and $\angle WYX$ is the middle sized angle in $\triangle WXY$ making $\angle W$ the largest angle. $m\angle WXY + m\angle W = m\angle Z + m\angle ZXY$ making $\angle WXY$ the smallest.

20. ★ **MULTIPLE CHOICE** Which group of side lengths can be used to construct a triangle? **B**

Ⓐ 3 yd, 4 ft, 5 yd Ⓑ 3 yd, 5 ft, 8 ft

Ⓒ 11 in., 16 in., 27 in. Ⓓ 2 ft, 11 in., 12 in.

B **POSSIBLE SIDE LENGTHS** ***Describe* the possible lengths of the third side of the triangle given the lengths of the other two sides.**

21. 5 inches, 12 inches **7 in. < *x* < 17 in.**

22. 3 meters, 4 meters **1 m < *x* < 7 m**

23. 12 feet, 18 feet **6 ft < *x* < 30 ft**

24. 10 yards, 23 yards **13 yd < *x* < 33 yd**

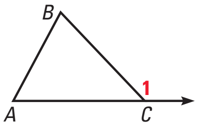
25. 2 feet, 40 inches **16 in. < *x* < 64 in.**

26. 25 meters, 25 meters **0 m < *x* < 50 m**

27. EXTERIOR ANGLE INEQUALITY Another triangle inequality relationship is given by the Exterior Angle Inequality Theorem. It states:

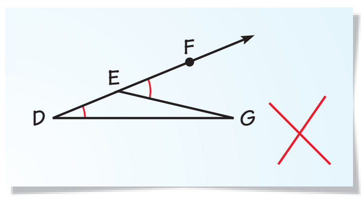
The measure of an exterior angle of a triangle is greater than the measure of either of the nonadjacent interior angles.

Use a relationship about triangle sum properties to *explain* how you know that $m\angle 1 > m\angle A$ and $m\angle 1 > m\angle B$ in $\triangle ABC$ with exterior angle $\angle 1$.

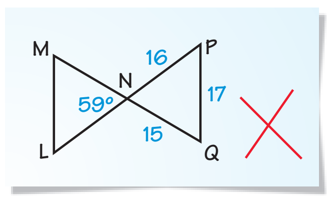


ERROR ANALYSIS **Use Theorems 5.10–5.12 and the theorem in Exercise 27 to *explain* why the diagram must be incorrect.**

28.



29.



The longest side is not opposite the largest angle.

30. ★ **SHORT RESPONSE** *Explain* why the hypotenuse of a right triangle must always be longer than either leg. **The hypotenuse is opposite the 90° angle in a right triangle which is the largest angle in the triangle.**

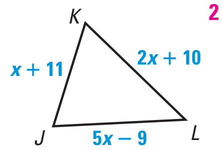
ORDERING MEASURES **Is it possible to build a triangle using the given side lengths? If so, list the angles of the triangle in order from least to greatest measure.**

31. $PQ = \sqrt{58}$, $QR = 2\sqrt{13}$, $PR = 5\sqrt{2}$ **yes; $\angle Q$, $\angle P$, $\angle R$**

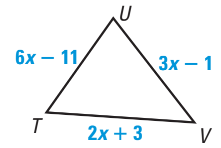
32. $ST = \sqrt{29}$, $TU = 2\sqrt{17}$, $SU = 13.9$ **no**

xy **ALGEBRA** ***Describe* the possible values of *x*.**

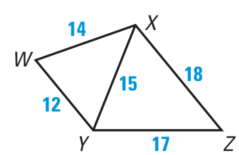
33. **$2 < x < 15$**



34. **$\frac{15}{7} < x < 13$**

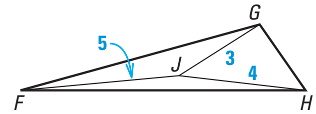


35. USING SIDE LENGTHS Use the diagram at the right. Suppose $\overline{XY}$ bisects $\angle WYZ$. List all six angles of $\triangle XYZ$ and $\triangle WXY$ in order from smallest to largest. *Explain* your reasoning.



C **36. CHALLENGE** The perimeter of $\triangle HGF$ must be between what two integers? *Explain* your reasoning.

$4 < P < 24$; $2 < FG < 8$, $1 < GH < 7$, and $1 < HF < 9$



◯ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE**

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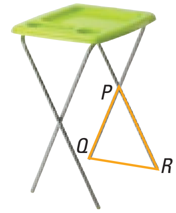
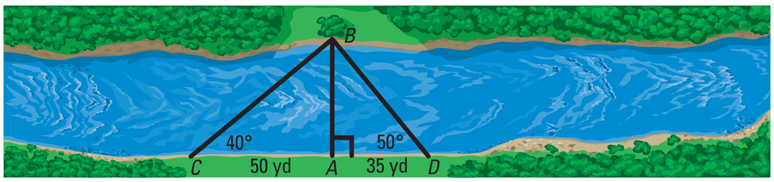
PROBLEM SOLVING

A **37. TRAY TABLE** In the tray table shown, $\overline{PQ} \cong \overline{PR}$ and $QR < PQ$. Write two inequalities about the angles in $\triangle PQR$. What other angle relationship do you know? **$m\angle P < m\angle Q$, $m\angle P < m\angle R$; $m\angle Q = m\angle R$**



38. INDIRECT MEASUREMENT You can estimate the width of the river at point A by taking several sightings to the tree across the river at point B. The diagram shows the results for locations C and D along the riverbank. Using $\triangle BCA$ and $\triangle BDA$, what can you conclude about AB, the width of the river at point A? What could you do if you wanted a closer estimate?



38. 35 yd < AB < 50 yd ***Sample answer:*** **Extend $\overrightarrow{BA}$ through E such that $m\angle ACE = 40°$, then measure $\overline{AE}$.**

EXAMPLE 3 for Ex. 39

39. ★ **EXTENDED RESPONSE** You are planning a vacation to Montana. You want to visit the destinations shown in the map.

a. A brochure states that the distance between Granite Peak and Fort Peck Lake is 1080 kilometers. *Explain* how you know that this distance is a misprint.

b. Could the distance from Granite Peak to Fort Peck Lake be 40 kilometers? *Explain*.

c. Write two inequalities to represent the range of possible distances from Granite Peak to Fort Peck Lake. **$d > 76$ km, $d < 1054$ km**

d. What can you say about the distance between Granite Peak and Fort Peck Lake if you know that $m\angle 2 < m\angle 1$ and $m\angle 2 < m\angle 3$? **The distance is less than 489 kilometers.**

39a. The sum of the other two side lengths is less than 1080.

39b. No; the sum of the distance from Granite Peak to Fort Peck Lake and Granite Peak to Glacier National Park must be more than 565.

FORMING TRIANGLES **In Exercises 40–43, you are given a 24 centimeter piece of string. You want to form a triangle out of the string so that the length of each side is a whole number. Draw figures accurately.**

B **40.** Can you decide if three side lengths form a triangle without checking all three inequalities shown for Theorem 5.12? If so, *describe* your shortcut. **Yes; pick the two shortest sides and see if their sum is greater than the third side.**

41. Draw four possible isosceles triangles and label each side length. Tell whether each of the triangles you formed is *acute*, *right*, or *obtuse*. **See margin.**

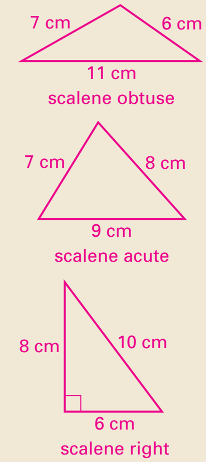
42. Draw three possible scalene triangles and label each side length. Try to form at least one scalene acute triangle and one scalene obtuse triangle. **See margin.**

43. List three combinations of side lengths that will not produce triangles. ***Sample answer:*** **3, 4, 17; 2, 5, 17; 4, 4, 16**

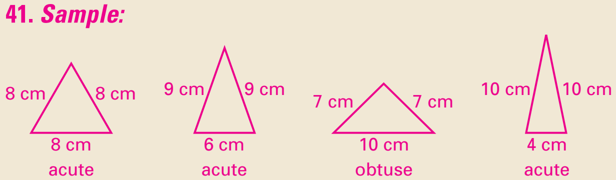
Internet Reference

Exercise 39 More information about Montana can be found at mt.gov

42. *Sample:*



41. *Sample:*



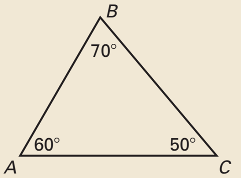
5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

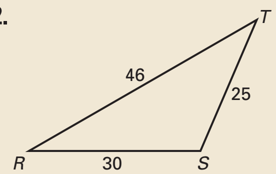
For Exercises 1 and 2, list the sides or angles in order from least to greatest.

1.



$\overline{AB}$, $\overline{BC}$, $\overline{AC}$

2.



$\angle R$, $\angle T$, $\angle S$

Tell whether the side lengths can form a triangle.

3. 37 m, 35 m, 18 m **yes**

4. 3 ft, 3 ft, 6 ft **no**

5. Jeremy wants to build a triangular toy using sticks. He has one stick that is 12 inches and another that is 10 inches. What are the possible lengths of the third side of the triangle? **greater than 2 in. and less than 22 in.**

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

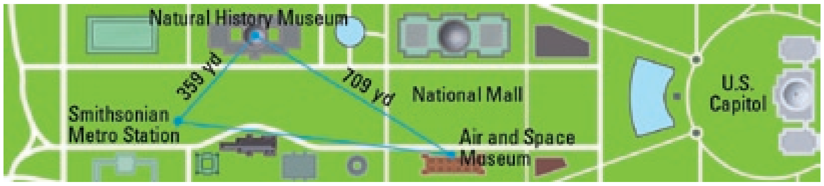
Challenge

Additional challenge is available in the Chapter Resource Book.

47, 48a–b. See Additional Answers.

45. $1\frac{1}{4}$ **mi** $\le d \le 2\frac{3}{4}$ **mi; if the locations are collinear then the distance could be** $1\frac{1}{4}$ **miles or** $2\frac{3}{4}$ **miles. If the locations are not collinear then the distance must be between** $1\frac{1}{4}$ **miles and** $2\frac{3}{4}$ **miles because of the Triangle Inequality Theorem.**

44. SIGHTSEEING You get off the Washington, D.C., subway system at the Smithsonian Metro station. First you visit the Museum of Natural History. Then you go to the Air and Space Museum. You record the distances you walk on your map as shown. *Describe* the range of possible distances you might have to walk to get back to the Smithsonian Metro station. **350 yd < d < 1068 yd**



45. ★ SHORT RESPONSE Your house is 2 miles from the library. The library is $\frac{3}{4}$ mile from the grocery store. What do you know about the distance from your house to the grocery store? *Explain*. Include the special case when the three locations are all in a straight line.

C **46. ISOSCELES TRIANGLES** For what combinations of angle measures in an isosceles triangle are the congruent sides shorter than the base of the triangle? longer than the base of the triangle?
vertex angle: 60° < x < 180°; vertex angle: 0° < x < 60°

47. PROVING THEOREM 5.12 Prove the Triangle Inequality Theorem. **See margin.**

GIVEN ▶ $\triangle ABC$

PROVE ▶ (1) $AB + BC > AC$
(2) $AC + BC > AB$
(3) $AB + AC > BC$

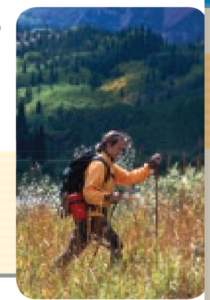
Plan for Proof One side, say $\overline{BC}$, is longer than or at least as long as each of the other sides. Then (1) and (2) are true. To prove (3), extend $\overline{AC}$ to D so that $\overline{AB} \cong \overline{AD}$ and use Theorem 5.11 to show that $DC > BC$.

48. CHALLENGE Prove the following statements. **See margin.**

a. The length of any one median of a triangle is less than half the perimeter of the triangle.

b. The sum of the lengths of the three medians of a triangle is greater than half the perimeter of the triangle.

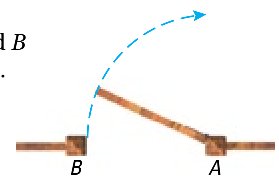
5.6 Inequalities in Two Triangles and Indirect Proof



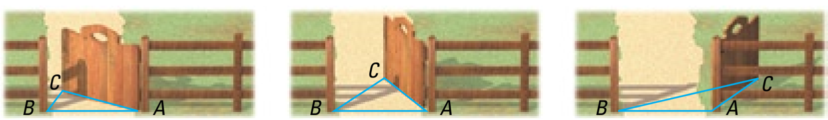
Before	You used inequalities to make comparisons in one triangle.
Now	You will use inequalities to make comparisons in two triangles.
Why?	So you can compare the distances hikers traveled, as in Ex. 22.

Key Vocabulary
- **indirect proof**
- **included angle**

Imagine a gate between fence posts A and B that has hinges at A and swings open at B.



As the gate swings open, you can think of $\triangle ABC$, with side $\overline{AC}$ formed by the gate itself, side $\overline{AB}$ representing the distance between the fence posts, and side $\overline{BC}$ representing the opening between post B and the outer edge of the gate.

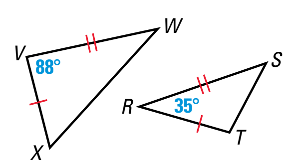


Notice that as the gate opens wider, both the measure of $\angle A$ and the distance CB increase. This suggests the *Hinge Theorem*.

THEOREMS — *For Your Notebook*

THEOREM 5.13 Hinge Theorem

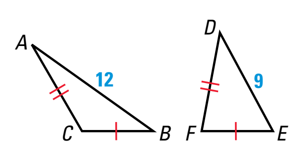
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.



$WX > ST$

THEOREM 5.14 Converse of the Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.



$m\angle C > m\angle F$

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. Write the if-then form, converse, inverse, and contrapositive of the given statement. $3x - 8 = 22$ because $x = 10$.
   If $x = 10$, then $3x - 8 = 22$.
   If $3x - 8 = 22$, then $x = 10$.
   If $x \neq 10$, then $3x - 8 \neq 22$.
   If $3x - 8 \neq 22$, then $x \neq 10$.
2. In $\triangle ABC$, $BC = 18$, $AB = 13$, and $AC = 16$. List the angles of the triangle from least to greatest. **$\angle C, \angle B, \angle A$**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block
- See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3

How do you write an indirect proof? **Tell students they will learn how to answer this question by using the opposite of what they are trying to prove.**

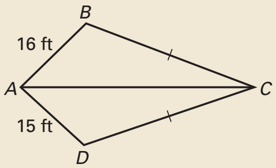
Motivating the Lesson

A snake opens its jaws to swallow food. Discuss how the angle formed by its jaws is related to the size of the food. Tell students that this lesson explores the relationship between an angle of a triangle and the length of the side opposite that angle.

3 TEACH

Extra Example 1

Given that $\overline{BC} \cong \overline{DC}$, how does $\angle ACB$ compare to $\angle ACD$?



$\angle ACB > \angle ACD$

Key Question to Ask for Example 1

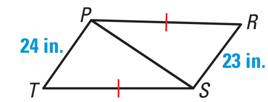
- What did you show in order to use the Converse of the Hinge Theorem. **Two pairs of corresponding sides are congruent and one third side is greater than the other third side.**

Animated Geometry
my.hrw.com

An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.

EXAMPLE 1 Use the Converse of the Hinge Theorem

Given that $\overline{ST} \cong \overline{PR}$, how does $\angle PST$ compare to $\angle SPR$?



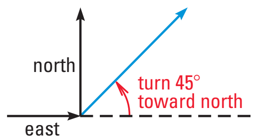
Solution

You are given that $\overline{ST} \cong \overline{PR}$ and you know that $\overline{PS} \cong \overline{PS}$ by the Reflexive Property. Because 24 inches > 23 inches, $PT > RS$. So, two sides of $\triangle STP$ are congruent to two sides of $\triangle PRS$ and the third side in $\triangle STP$ is longer.

▶ By the Converse of the Hinge Theorem, $m\angle PST > m\angle SPR$.

EXAMPLE 2 Solve a multi-step problem

BIKING Two groups of bikers leave the same camp heading in opposite directions. Each group goes 2 miles, then changes direction and goes 1.2 miles. Group A starts due east and then turns 45° toward north as shown. Group B starts due west and then turns 30° toward south.



Which group is farther from camp? Explain your reasoning.

Solution

Draw a diagram and mark the given measures. The distances biked and the distances back to camp form two triangles, with congruent 2 mile sides and congruent 1.2 mile sides. Add the third sides of the triangles to your diagram.

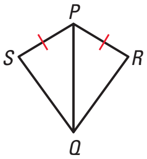
Next use linear pairs to find and mark the included angles of 150° and 135°.

▶ Because 150° > 135°, Group B is farther from camp by the Hinge Theorem.

Animated Geometry at my.hrw.com

✓ GUIDED PRACTICE for Examples 1 and 2

Use the diagram at the right.



1. If $PR = PS$ and $m\angle QPR > m\angle QPS$, which is longer, $\overline{SQ}$ or $\overline{RQ}$? **$\overline{RQ}$**
2. If $PR = PS$ and $RQ < SQ$, which is larger, $\angle RPQ$ or $\angle SPQ$? **$\angle SPQ$**
3. **WHAT IF?** In Example 2, suppose Group C leaves camp and goes 2 miles due north. Then they turn 40° toward east and continue 1.2 miles. *Compare* the distances from camp for all three groups. **Group *B* is the farthest from camp, followed by Group *C*, and then Group *A* which is the closest.**

INDIRECT REASONING Suppose a student looks around the cafeteria, concludes that hamburgers are not being served, and explains as follows.

> *At first I assumed that we are having hamburgers because today is Tuesday and Tuesday is usually hamburger day.*
>
> *There is always ketchup on the table when we have hamburgers, so I looked for the ketchup, but I didn't see any.*
>
> *So, my assumption that we are having hamburgers must be false.*

The student used *indirect* reasoning. So far in this book, you have reasoned *directly* from given information to prove desired conclusions.

In an **indirect proof**, you start by making the temporary assumption that the desired conclusion is false. By then showing that this assumption leads to a logical impossibility, you prove the original statement true *by contradiction*.

KEY CONCEPT — *For Your Notebook*

How to Write an Indirect Proof (Proof by Contradiction)

STEP 1 **Identify** the statement you want to prove. **Assume** temporarily that this statement is false by assuming that its opposite is true.

STEP 2 **Reason** logically until you reach a contradiction.

STEP 3 **Point out** that the desired conclusion must be true because the contradiction proves the temporary assumption false.

EXAMPLE 3 Write an indirect proof

Write an indirect proof that an odd number is not divisible by 4.

GIVEN ▶ x is an odd number.

PROVE ▶ x is not divisible by 4.

Solution

STEP 1 Assume temporarily that x is divisible by 4. This means that $\frac{x}{4} = n$ for some whole number n. So, multiplying both sides by 4 gives $x = 4n$.

STEP 2 If x is odd, then, by definition, x cannot be divided evenly by 2. However, $x = 4n$ so $\frac{x}{2} = \frac{4n}{2} = 2n$. We know that $2n$ is a whole number because n is a whole number, so x *can* be divided evenly by 2. This contradicts the given statement that x is odd.

STEP 3 Therefore, the assumption that x is divisible by 4 must be false, which proves that x is not divisible by 4.

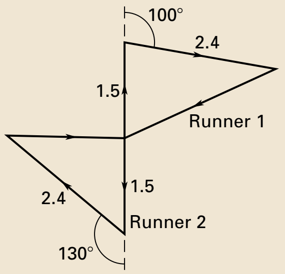
READ VOCABULARY
You have reached a *contradiction* when you have two statements that cannot both be true at the same time.

✓ GUIDED PRACTICE for Example 3

4. Suppose you wanted to prove the statement "If $x + y \neq 14$ and $y = 5$, then $x \neq 9$." What temporary assumption could you make to prove the conclusion indirectly? How does that assumption lead to a contradiction?
Assume temporarily that $x = 9$; since $x + y \neq 14$ and $y = 5$ are given, letting $x = 9$ leads to the contradiction $9 + 5 \neq 14$.

Extra Example 2

Two runners start together and run in opposite directions. Each one goes 1.5 miles, changes direction, and goes 2.4 miles. The first runner starts due north and turns 100° towards the east. The other runner starts due south and turns 130° towards the west. Both runners return to the starting point. Which runner ran farther? *Explain.*



Each triangle has side lengths 1.5 mi and 2.4 mi, and the angles between those sides are 80° and 50°. By the Hinge Theorem, the third side of the triangle for Runner 1 is longer, so Runner 1 ran further.

Extra Example 3

Write an indirect proof to show that the sum of two odd numbers is even.

Given a and b are odd numbers.

Prove $a + b$ is an even number.

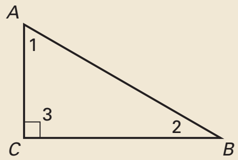
Step 1. Assume $a + b$ is not even, which is the same as assuming that $a + b$ is odd.

Step 2. If a and b are odd, then $a = 2m + 1$ and $b = 2n + 1$ for some m and n. Then $a + b = (2m + 1) + (2n + 1) = 2(m + n) + 2$, which is an even number. This contradicts the assumption that $a + b$ is odd.

Step 3. The assumption that $a + b$ is odd must be false. Therefore, $a + b$ must be an even number.

Extra Example 4

Write an indirect proof of the Corollary of the Triangle Sum Theorem, page 220, that the acute angles of a right triangle are complementary.



Given $\angle C$ is a right angle.

Prove $m\angle 1 + m\angle 2 = 90°$

Assume temporarily that $m\angle 1 + m\angle 2 \neq 90°$.

Case 1. If $m\angle 1 + m\angle 2 < 90°$, then $m\angle 3 > 90°$ because of the Triangle Sum Theorem. But this contradicts the statement that $\angle C$ is a right angle.

Case 2. If $m\angle 1 + m\angle 2 > 90°$, then $m\angle 3 < 90°$ because of the Triangle Sum Theorem. Again, this contradicts the statement that $\angle C$ is a right angle.

Therefore, $m\angle 1 + m\angle 2 = 90°$.

Key Question to Ask for Example 4

- Why are there two cases in the proof? **If $m\angle B$ is not greater than $m\angle E$, then there are two possibilities: $m\angle B = m\angle E$ or $m\angle B < m\angle E$.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you organize an indirect proof?

- **If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second, and vice versa.**
- **To write an indirect proof, start by assuming the opposite of what you want to prove and then reason logically until you reach a contradiction.**

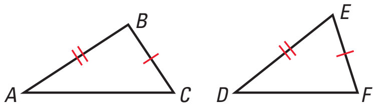
Assume the opposite of what you are trying to prove is true and then reason logically until you reach a contradiction.

EXAMPLE 4 Prove the Converse of the Hinge Theorem

Write an indirect proof of the Converse of the Hinge Theorem.

GIVEN ▶ $\overline{AB} \cong \overline{DE}$
$\overline{BC} \cong \overline{EF}$
$AC > DF$

PROVE ▶ $m\angle B > m\angle E$



Proof Assume temporarily that $m\angle B \ngtr m\angle E$. Then, it follows that either $m\angle B = m\angle E$ or $m\angle B < m\angle E$.

Case 1 If $m\angle B = m\angle E$, then $\angle B \cong \angle E$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate and $AC = DF$.

Case 2 If $m\angle B < m\angle E$, then $AC < DF$ by the Hinge Theorem.

Both conclusions contradict the given statement that $AC > DF$. So, the temporary assumption that $m\angle B \ngtr m\angle E$ cannot be true. This proves that $m\angle B > m\angle E$.

✓ GUIDED PRACTICE for Example 4

5. Write a temporary assumption you could make to prove the Hinge Theorem indirectly. What two cases does that assumption lead to?
The third side of the first is less than or equal to the third side of the second; Case 1: Third side of the first equals the third side of the second. Case 2: Third side of the first is less than the third side of the second.

5.6 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 7, and 23
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 19, and 25

SKILL PRACTICE

A

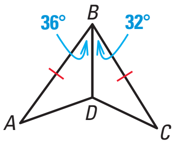
1. **VOCABULARY** Why is indirect proof also called *proof by contradiction*?
You temporarily assume that the desired conclusion is false and this leads to a logical contradiction.

2. ★ **WRITING** *Explain* why the name "Hinge Theorem" is used for Theorem 5.13. ***Sample answer:*** **Consider opening a door; the wider you open it, the bigger the angle the door makes with the wall and the larger the opening.**

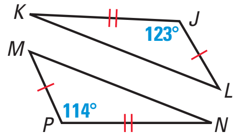
EXAMPLE 1 for Exs. 3–10

APPLYING THEOREMS **Copy and complete with <, >, or =. *Explain.*** **3–8. See margin for explanations.**

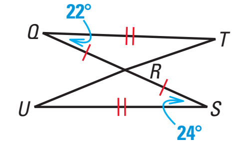
3. AD _?_ CD **>**



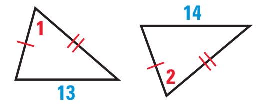
4. MN _?_ LK **<**



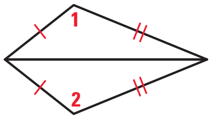
5. TR _?_ UR **<**



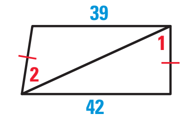
6. $m\angle 1$ _?_ $m\angle 2$ **<**



7. $m\angle 1$ _?_ $m\angle 2$ **=**

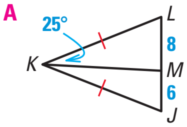


8. $m\angle 1$ _?_ $m\angle 2$ **>**



3. $AD > CD$ by the Hinge Theorem, because $m\angle ABD > m\angle CBD$.
4. $MN < LK$ by the Hinge Theorem, because $m\angle MPN < m\angle LJK$.
5. $TR < UR$ by the Hinge Theorem, because $m\angle TQR < m\angle USR$.
6. $m\angle 1 < m\angle 2$ by the Converse of the Hinge Theorem, because $13 < 14$.
7. $m\angle 1 = m\angle 2$ because the triangles are congruent by the SSS Congruence Postulate, so corresponding parts $\angle 1$ and $\angle 2$ are congruent.
8. $m\angle 1 > m\angle 2$ by the Converse of the Hinge Theorem, because $42 > 39$.

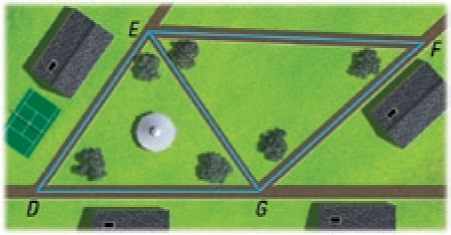
9. ★ **MULTIPLE CHOICE** Which is a possible measure for $\angle JKM$? **A**



Ⓐ 20° Ⓑ 25°

Ⓒ 30° Ⓓ Cannot be determined

10. **USING A DIAGRAM** The path from E to F is longer than the path from E to D. The path from G to D is the same length as the path from G to F. What can you conclude about the angles of the paths? *Explain* your reasoning. ***m∠DGE < m∠FGE*; Converse of the Hinge Theorem applies.**



EXAMPLES B 3 and 4 for Exs. 11–13

STARTING AN INDIRECT PROOF **In Exercises 11 and 12, write a temporary assumption you could make to prove the conclusion indirectly.**

11. If x and y are odd integers, then xy is odd. **Suppose *xy* is even.**

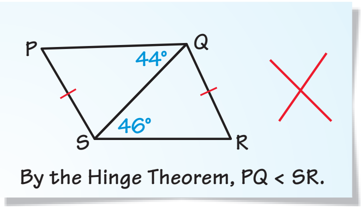
12. In $\triangle ABC$, if $m\angle A = 100°$, then $\angle B$ is not a right angle. **Suppose ∠*B* is a right angle.**

13. **REASONING** Your study partner is planning to write an indirect proof to show that $\angle A$ is an obtuse angle. She states "Assume temporarily that $\angle A$ is an acute angle." What has your study partner overlooked? **∠*A* could be a right angle or a straight angle.**

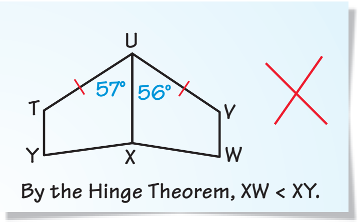
ERROR ANALYSIS ***Explain* why the student's reasoning is not correct.**

14. To use the Hinge Theorem the angle must be the included angle between the two pairs of congruent sides.

14.



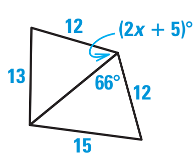
15.



The Hinge Theorem is about triangles not quadrilaterals.

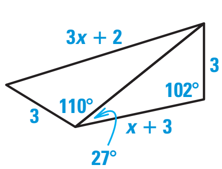
xy **ALGEBRA** **Use the Hinge Theorem or its converse and properties of triangles to write and solve an inequality to describe a restriction on the value of *x*.**

16.



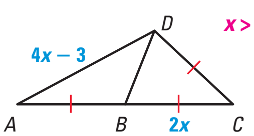
$x < 30\frac{1}{2}$

17.



$x > \frac{1}{2}$

18.



$x > \frac{3}{2}$

C **20. In △*FHG*, *m∠H* > *m∠GFH* or *m∠FGH* because the side opposite the larger angle is longer than the side opposite the smaller angle.**

19. ★ **SHORT RESPONSE** If $\overline{NR}$ is a median of $\triangle NPQ$ and $NQ > NP$, *explain* why $\angle NRQ$ is obtuse. **Using the Converse of the Hinge Theorem ∠*NRQ* > ∠*NRP*. Since ∠*NRQ* and ∠*NRP* are a linear pair ∠*NRQ* must be obtuse and ∠*NRP* must be acute.**

20. **ANGLE BISECTORS** In $\triangle EFG$, the bisector of $\angle F$ intersects the bisector of $\angle G$ at point H. *Explain* why $\overline{FG}$ must be longer than $\overline{FH}$ or $\overline{HG}$.

21. **CHALLENGE** In $\triangle ABC$, the altitudes from B and C meet at D. What is true about $\triangle ABC$ if $m\angle BAC > m\angle BDC$? *Justify* your answer. **△*ABC* is obtuse; the orthocenter of an obtuse triangle is always outside of the triangle, making the angle smaller outside than inside.**

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–10, 22
Day 2:
Exs. 11–14, 23–25

Average:
Day 1:
Exs. 1–10, 16–18, 22
Day 2:
Exs. 11–15, 19, 23–26

Advanced:
Day 1:
Exs. 1–10, 16–18, 22
Day 2:
Exs. 11–15, 19–21*, 23–28*

Block:
Exs. 1–19, 22–26

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 10, 11, 22, 23
Average: 6, 12, 16, 22, 24
Advanced: 8, 13, 17, 22, 24

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

Avoiding Common Errors

Exercises 11–12 A common error is to assume that the opposite of the *given* is false. Caution students to assume the opposite of the *conclusion* is false.

Teaching Strategy

Exercise 26 Point out that this proof depends on knowing that the largest angle of a right triangle is the right angle. Exercise 26 can also be proved indirectly by using the Triangle Sum Theorem.

Mathematical Reasoning

Exercise 27 Using symbolic notation, the contrapositive of the conditional $p \rightarrow q$ is $\sim q \rightarrow \sim p$. If we know that both $\sim q \rightarrow \sim p$ and $\sim q$ are true, then the Law of Detachment lets us conclude that $\sim p$ must be true. If p is given, this is a contradiction because p and $\sim p$ cannot both be true.

Study Strategy

Exercise 28 Encourage students to list the key points of the plan for the proof of the Hinge Theorem. Then they can prove each statement in their list.

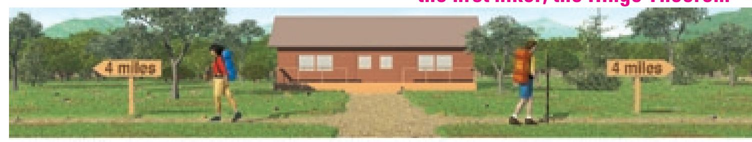
25c. ***Sample answer:*** **Since $NL = NK = NM$ and as $m\angle LNK$ increases KL increases, and $m\angle KNM$ decreases as KM decreases, you have two pairs of congruent sides with $m\angle LNK$ eventually greater than $m\angle KNM$. The Hinge Theorem guarantees KL will eventually be greater than KM.**

26. ***Sample answer:*** **Assume temporarily that $\overline{AB}$ is not the shortest segment from A to k. This implies that there is a point C on k such that $\overline{AC}$ is the shortest segment. $\triangle ABC$ is a right triangle with hypotenuse $\overline{AC}$. Since $\overline{AC}$ is opposite the right angle, the largest angle in a right triangle, Theorem 5.10 guarantees that $\overline{AC}$ is the longest side. This contradicts the assumption that $\overline{AC}$ is the shortest side, making $\overline{AB}$ the shortest side.**

PROBLEM SOLVING

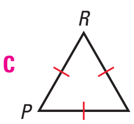
EXAMPLE 2 A for Ex. 22

22. HIKING Two hikers start at the visitor center. The first hikes 4 miles due west, then turns 40° toward south and hikes 1.8 miles. The second hikes 4 miles due east, then turns 52° toward north and and hikes 1.8 miles. Which hiker is farther from the visitor center? *Explain* how you know. **the first hiker; the Hinge Theorem**



EXAMPLES 3 and 4 for Exs. 23–24

23. INDIRECT PROOF Arrange statements A–E in order to write an indirect proof of the corollary: If $\triangle PQR$ is *equilateral*, then it is *equiangular*. **E, A, D, B, C**



GIVEN ▶ $\triangle PQR$ is equilateral.

A. That means that for some pair of vertices, say P and Q, $m\angle P > m\angle Q$.

B. But this contradicts the given statement that $\triangle PQR$ is equilateral.

C. The contradiction shows that the temporary assumption that $\triangle PQR$ is not equiangular is false. This proves that $\triangle PQR$ is equiangular.

D. Then, by Theorem 5.11, you can conclude that $QR > PR$.

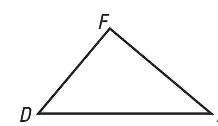
E. Temporarily assume that $\triangle PQR$ is not equiangular.

24. PROVING THEOREM 5.11 Write an indirect proof of Theorem 5.11: If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

GIVEN ▶ $m\angle D > m\angle E$

PROVE ▶ $EF > DF$

Plan for Proof In Case 1, assume that $EF < DF$. In Case 2, assume that $EF = DF$.



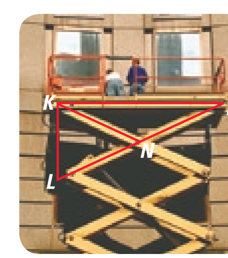
24. Assume B **temporarily that either $EF < DF$ or $EF = DF$. Case 1: Assume $EF < DF$. Using Theorem 5.10 you would get $m\angle D < m\angle E$ which contradicts the given statement. Case 2: Assume $EF = DF$. If $EF = DF$ then $\triangle FDE$ would be isosceles with $m\angle D = m\angle E$ which contradicts the given statement. Therefore, the assumption must be false. So, $EF > DF$.**

25. ★ EXTENDED RESPONSE A scissors lift can be used to adjust the height of a platform.

a. Interpret As the mechanism expands, $\overline{KL}$ gets longer. As KL increases, what happens to $m\angle LNK$? to $m\angle KNM$? **It gets larger; it gets smaller.**

b. Apply Name a distance that decreases as $\overline{KL}$ gets longer. **KM**

c. Writing *Explain* how the adjustable mechanism illustrates the Hinge Theorem. **See margin.**

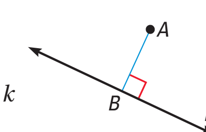


26. PROOF Write a proof that the shortest distance from a point to a line is the length of the perpendicular segment from the point to the line. **See margin.**

GIVEN ▶ Line k; point A not on k; point B on k such that $\overline{AB} \perp k$

PROVE ▶ $\overline{AB}$ is the shortest segment from A to k.

Plan for Proof Assume that there is a shorter segment from A to k and use Theorem 5.10 to show that this leads to a contradiction.



Robert Brenner/PhotoEdit

○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

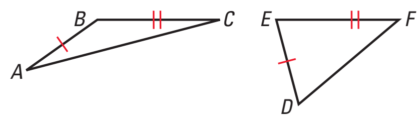
27. Prove: If x is divisible by 4, then x is even. Proof: Since x is divisible by 4, $x = 4a$. When you factor out a 2, you get $x = 2(2a)$ which is in the form $2n$, which implies x is an even number; your temporary assumption in the indirect proof is the same as your hypothesis in the direct proof.

C **27. USING A CONTRAPOSITIVE** Because the contrapositive of a conditional is equivalent to the original statement, you can prove the statement by proving its contrapositive. Look back at the conditional in Example 3. Write a proof of the contrapositive that uses direct reasoning. How is your proof similar to the indirect proof of the original statement? **See margin.**

28. CHALLENGE Write a proof of the Hinge Theorem. **See margin.**

GIVEN ▶ $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $m\angle ABC > m\angle DEF$

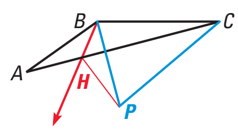
PROVE ▶ $AC > DF$



Plan for Proof

1. Because $m\angle ABC > m\angle DEF$, you can locate a point P in the interior of $\angle ABC$ so that $\angle CBP \cong \angle FED$ and $\overline{BP} \cong \overline{ED}$. Draw $\overline{BP}$ and show that $\triangle PBC \cong \triangle DEF$.
2. Locate a point H on $\overline{AC}$ so that $\overrightarrow{BH}$ bisects $\angle PBA$ and show that $\triangle ABH \cong \triangle PBH$.
3. Give reasons for each statement below to show that $AC > DF$.

$AC = AH + HC = PH + HC > PC = DF$

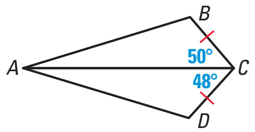


QUIZ

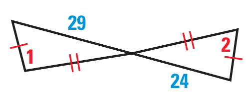
1. Is it possible to construct a triangle with side lengths 5, 6, and 12? If not, *explain* why not. **No; 5 + 6 must be greater than 12.**
2. The lengths of two sides of a triangle are 15 yards and 27 yards. *Describe* the possible lengths of the third side of the triangle. **12 yd < *x* < 42 yd**
3. In $\triangle PQR$, $m\angle P = 48°$ and $m\angle Q = 79°$. List the sides of $\triangle PQR$ in order from shortest to longest. **$\overline{QR}$, $\overline{PQ}$, $\overline{PR}$**

Copy and complete with <, >, or =.

4. BA __?__ DA **>**



5. $m\angle 1$ __?__ $m\angle 2$ **>**



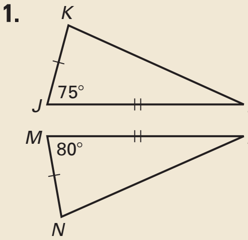
5 ASSESS AND RETEACH

Daily Homework Quiz

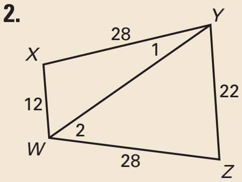
Also available online

Complete each statement with <, >, or =.

1. KL __?__ NP **<**



2. $m\angle 1$ __?__ $m\angle 2$ **<**



3. Suppose you want to write an indirect proof of this statement: "In $\triangle ABC$, if $m\angle A > 90°$ then $\triangle ABC$ is not a right triangle." What temporary assumption should start your proof? **Assume $\triangle ABC$ is a right triangle.**

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

28. See Additional Answers.

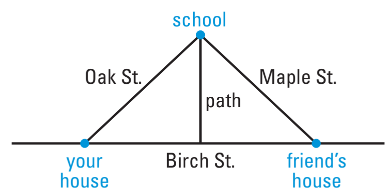
2. Dawson. *Sample answer:* The Hinge Theorem guarantees that Allentown to Dawson is the shortest distance since the included angle is 120°, unlike Allentown to Bakersville where the included angle is 145°.

4. No. *Sample answer:* In a triangle the largest angle should be opposite the longest side. The side measuring 13.55 centimeters is opposite the right angle yet 13.7 centimeters is the longest side in the right triangle on the left.

5c. 24 ft by 16 ft by 32 ft; since two of the sides are 24 feet and 16 feet, the third side must be 32 feet so the dog can run at least 25 feet within the pen.

MIXED REVIEW *of Problem Solving*

1. MULTI-STEP PROBLEM In the diagram below, the entrance to the path is halfway between your house and your friend's house.

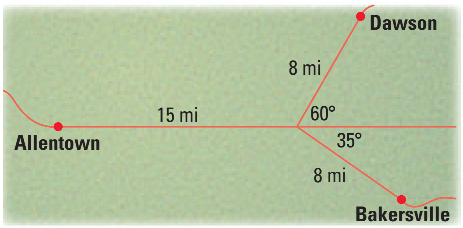


a. Can you conclude that you and your friend live the same distance from the school if the path bisects the angle formed by Oak and Maple Streets? **no**

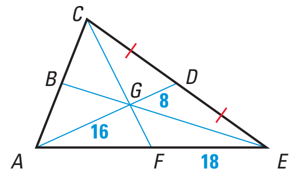
b. Can you conclude that you and your friend live the same distance from the school if the path is perpendicular to Birch Street? **yes**

c. Your answers to parts (a) and (b) show that a triangle must be isosceles if which two special segments are equal in length? **altitude and median**

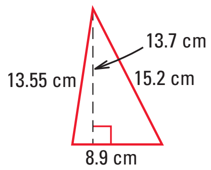
2. SHORT RESPONSE The map shows your driving route from Allentown to Bakersville and from Allentown to Dawson. Which city, Bakersville or Dawson, is located closer to Allentown? *Explain* your reasoning. **See margin.**



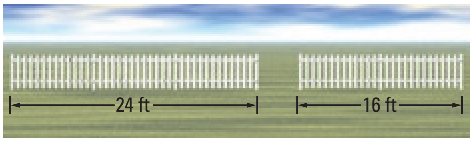
3. GRIDDED RESPONSE Find the length of $\overline{AF}$. **18**



4. SHORT RESPONSE In the instructions for creating the terrarium shown, you are given a pattern for the pieces that form the roof. Does the diagram for the red triangle appear to be correct? *Explain* why or why not. **See margin.**



5. EXTENDED RESPONSE You want to create a triangular fenced pen for your dog. You have the two pieces of fencing shown, so you plan to move those to create two sides of the pen.

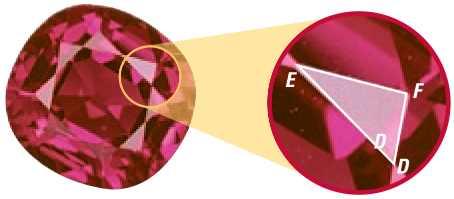


a. *Describe* the possible lengths for the third side of the pen. **8 ft < ℓ < 40 ft**

b. The fencing is sold in 8 foot sections. If you use whole sections, what lengths of fencing are possible for the third side? **16 ft, 24 ft, 32 ft**

c. You want your dog to have a run within the pen that is at least 25 feet long. Which pen(s) could you use? *Explain.* **See margin.**

6. OPEN-ENDED In the gem shown, give a possible side length of $\overline{DE}$ if $m\angle EFD > 90°$, $DF = 0.4$ mm, and $EF = 0.63$ mm. **0.63 mm < DE < 1.03 mm**



BIG IDEAS

For Your Notebook

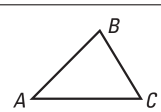
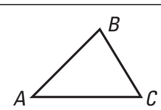
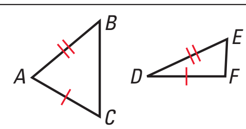
Big Idea 1

Using Properties of Special Segments in Triangles

Special segment	***Properties to remember***
Midsegment	Parallel to side opposite it and half the length of side opposite it
Perpendicular bisector	Concurrent at the circumcenter, which is: • equidistant from 3 vertices of △ • center of *circumscribed* circle that passes through 3 vertices of △
Angle bisector	Concurrent at the incenter, which is: • equidistant from 3 sides of △ • center of *inscribed* circle that just touches each side of △
Median (connects vertex to midpoint of opposite side)	Concurrent at the centroid, which is: • located two thirds of the way from vertex to midpoint of opposite side • balancing point of △
Altitude (perpendicular to side of △ through opposite vertex)	Concurrent at the orthocenter Used in finding area: If b is length of any side and h is length of altitude to that side, then $A = \frac{1}{2}bh$.

Big Idea 2

Using Triangle Inequalities to Determine What Triangles are Possible

Sum of lengths of any two sides of a △ is greater than length of third side.		$AB + BC > AC$ $AB + AC > BC$ $BC + AC > AB$
In a △, longest side is opposite largest angle and shortest side is opposite smallest angle.		If $AC > AB > BC$, then $m\angle B > m\angle C > m\angle A$. If $m\angle B > m\angle C > m\angle A$, then $AC > AB > BC$.
If two sides of a △ are ≅ to two sides of another △, then the △ with longer third side also has larger included angle.		If $BC > EF$, then $m\angle A > m\angle D$. If $m\angle A > m\angle D$, then $BC > EF$.

Big Idea 3

Extending Methods for Justifying and Proving Relationships

Coordinate proof uses the coordinate plane and variable coordinates. *Indirect proof* involves assuming the conclusion is false and then showing that the assumption leads to a contradiction.

Additional Resources

The following resources are available to help review the materials in this chapter.

Chapter Resource Book

- Chapter Review Games and Activities
- Cumulative Practice

Student Resources in Spanish

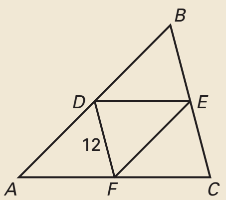
@HomeTutor

Vocabulary Practice

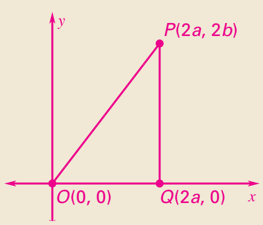
Vocabulary practice is available at **my.hrw.com**

Extra Example 1

In the diagram, $\overline{DF}$ is a midsegment of $\triangle ABC$. Find BC. **24**



8.



5 CHAPTER REVIEW

@HomeTutor
my.hrw.com
- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see p. PT2.

- midsegment of a triangle
- coordinate proof
- perpendicular bisector
- equidistant
- concurrent
- point of concurrency
- circumcenter
- incenter
- median of a triangle
- centroid
- altitude of a triangle
- orthocenter
- indirect proof

VOCABULARY EXERCISES

1. Copy and complete: A __?__ is a segment, ray, line, or plane that is perpendicular to a segment at its midpoint. **perpendicular bisector**

2. **WRITING** *Explain* how to draw a circle that is circumscribed about a triangle. What is the center of the circle called? *Describe* its radius.

2. Find the intersection of three perpendicular bisectors of the triangle. Using this point as the center of the circle, draw a circle whose radius is the distance from the point to any of the vertices; circumcenter; the distance from the circumcenter to any of the vertices of the triangle.

In Exercises 3–5, match the term with the correct definition.

3. Incenter **B**
4. Centroid **A**
5. Orthocenter **C**

A. The point of concurrency of the medians of a triangle
B. The point of concurrency of the angle bisectors of a triangle
C. The point of concurrency of the altitudes of a triangle

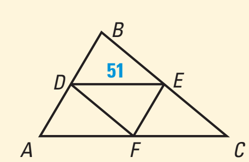
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of this chapter.

5.1 Midsegment Theorem and Coordinate Proof

EXAMPLE

In the diagram, $\overline{DE}$ is a midsegment of $\triangle ABC$. Find AC.



By the Midsegment Theorem, $DE = \frac{1}{2}AC$.

So, $AC = 2DE = 2(51) = 102$.

EXERCISES

EXAMPLES 1, 4, and 5 for Exs. 6–8

Use the diagram above where $\overline{DF}$ and $\overline{EF}$ are midsegments of $\triangle ABC$.

6. If $AB = 72$, find EF. **36**

7. If $DF = 45$, find EC. **45**

8. Graph $\triangle PQO$, with vertices $P(2a, 2b)$, $Q(2a, 0)$, and $O(0, 0)$. Find the coordinates of midpoint S of $\overline{PQ}$ and midpoint T of $\overline{QO}$. Show $\overline{ST} \parallel \overline{PO}$.
$S(2a, b)$, $T(a, 0)$; slope of $\overline{ST}$ is $\frac{b}{a}$ and slope of $\overline{OP}$ is $\frac{b}{a}$; see margin for art.

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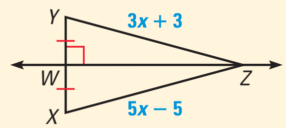
Chapter Review Practice

5.2 Use Perpendicular Bisectors

EXAMPLE

Use the diagram at the right to find XZ.



$\overleftrightarrow{WZ}$ is the perpendicular bisector of $\overline{XY}$.

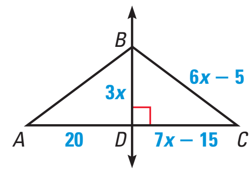
$5x - 5 = 3x + 3$	**By the Perpendicular Bisector Theorem, $ZX = ZY$.**
$x = 4$	**Solve for x.**

▶ So, $XZ = 5x - 5 = 5(4) - 5 = 15$.

EXERCISES

EXAMPLES 1 and 2 for Exs. 9–11

In the diagram, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.



9. What segment lengths are equal? ***BA* and *BC*, *DA* and *DC***

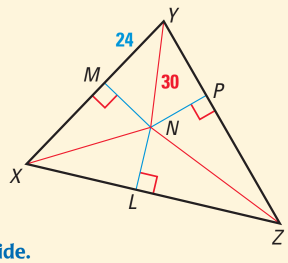
10. What is the value of x? **5**

11. Find AB. **25**

5.3 Use Angle Bisectors of Triangles

EXAMPLE

In the diagram, N is the incenter of $\triangle XYZ$. Find NL.



Use the Pythagorean Theorem to find NM in $\triangle NMY$.

$c^2 = a^2 + b^2$	**Pythagorean Theorem**
$30^2 = NM^2 + 24^2$	**Substitute known values.**
$900 = NM^2 + 576$	**Multiply.**
$324 = NM^2$	**Subtract 576 from each side.**
$18 = NM$	**Take positive square root of each side.**

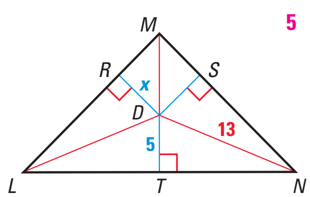
▶ By the Concurrency of Angle Bisectors of a Triangle, the incenter N of $\triangle XYZ$ is equidistant from all three sides of $\triangle XYZ$. So, because $NM = NL$, $NL = 18$.

EXERCISES

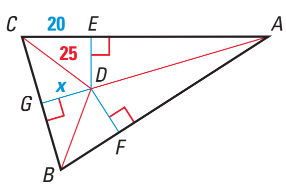
EXAMPLE 4 for Exs. 12–13

Point D is the incenter of the triangle. Find the value of x.

12. **5**

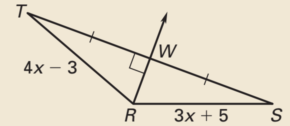


13. **15**



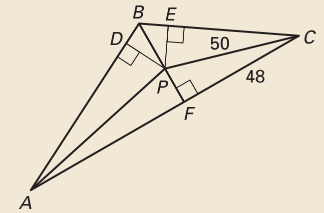
Extra Example 2

Uses the diagram to find RS. **29**



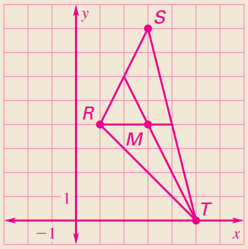
Extra Example 3

In the diagram, P is the incenter of $\triangle ABC$. Find PD. **14**



Extra Example 4

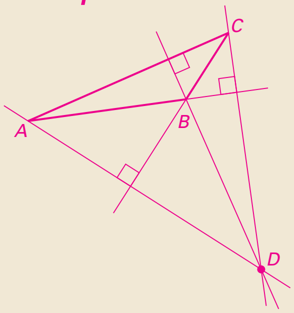
The vertices of $\triangle RST$ are $R(1, 4)$, $S(3, 8)$, and $T(5, 0)$. Find the coordinates of the centroid, M. **(3, 4)**



Extra Example 5

A triangle has one side of length 13 and another of length 18. Describe the possible lengths of the third side. **The length of the third side must be greater than 5 and less than 31.**

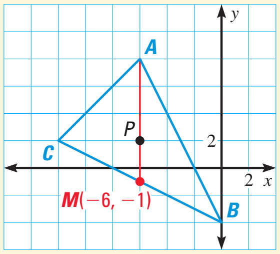
18. *Sample:*



5.4 Use Medians and Altitudes

EXAMPLE

The vertices of $\triangle ABC$ are $A(-6, 8)$, $B(0, -4)$, and $C(-12, 2)$. Find the coordinates of its centroid P.



Sketch $\triangle ABC$. Then find the midpoint M of $\overline{BC}$ and sketch median $\overline{AM}$.

$$M\left(\frac{-12+0}{2}, \frac{2+(-4)}{2}\right) = M(-6, -1)$$

The centroid is two thirds of the distance from a vertex to the midpoint of the opposite side.

The distance from vertex $A(-6, 8)$ to midpoint $M(-6, -1)$ is $8 - (-1) = 9$ units.

So, the centroid P is $\frac{2}{3}(9) = 6$ units down from A on $\overline{AM}$.

▸ The coordinates of the centroid P are $(-6, 8 - 6)$, or $(-6, 2)$.

EXERCISES

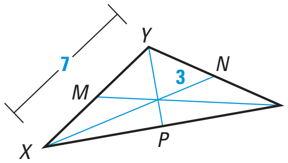
EXAMPLES 1, 2, and 3 for Exs. 14–18

Find the coordinates of the centroid D of $\triangle RST$.

14. $R(-4, 0)$, $S(2, 2)$, $T(2, -2)$ **(0, 0)**

15. $R(-6, 2)$, $S(-2, 6)$, $T(2, 4)$ **(−2, 4)**

Point Q is the centroid of $\triangle XYZ$.



16. Find XQ. **6**

17. Find XM. **3.5**

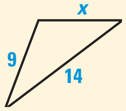
18. Draw an obtuse $\triangle ABC$. Draw its three altitudes. Then label its orthocenter D. **See margin.**

5.5 Use Inequalities in a Triangle

EXAMPLE

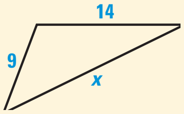
A triangle has one side of length 9 and another of length 14. Describe the possible lengths of the third side.

Let x represent the length of the third side. Draw diagrams and use the Triangle Inequality Theorem to write inequalities involving x.



$x + 9 > 14$

$x > 5$



$9 + 14 > x$

$23 > x$, or $x < 23$

▸ The length of the third side must be greater than 5 and less than 23.

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EXERCISES

EXAMPLES 1, 2, and 3 for Exs. 19–24

***Describe* the possible lengths of the third side of the triangle given the lengths of the other two sides.**

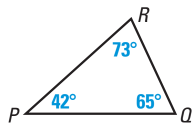
19. 4 inches, 8 inches
4 in. $< \ell <$ 12 in.

20. 6 meters, 9 meters
3 m $< \ell <$ 15 m

21. 12 feet, 20 feet
8 ft $< \ell <$ 32 ft

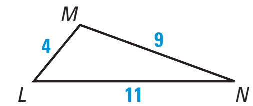
List the sides and the angles in order from smallest to largest.

22.



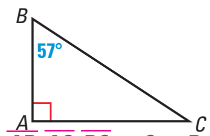
$\overline{RQ}, \overline{PR}, \overline{QP}; \angle P, \angle Q, \angle R$

23.



$\overline{LM}, \overline{MN}, \overline{LN}; \angle N, \angle L, \angle M$

24.

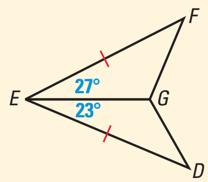


$\overline{AB}, \overline{AC}, \overline{BC}; \angle C, \angle B, \angle A$

5.6 Inequalities in Two Triangles and Indirect Proof

EXAMPLE

How does the length of $\overline{DG}$ compare to the length of $\overline{FG}$?



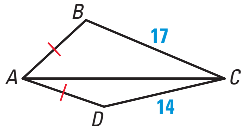
Because 27° > 23°, $m\angle GEF > m\angle GED$. You are given that $\overline{DE} \cong \overline{FE}$ and you know that $\overline{EG} \cong \overline{EG}$. Two sides of $\triangle GEF$ are congruent to two sides of $\triangle GED$ and the included angle in $\triangle GEF$ is larger so, by the Hinge Theorem, $FG > DG$.

EXERCISES

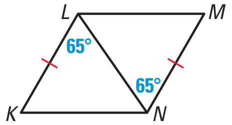
EXAMPLES 1, 3, and 4 for Exs. 25–27

Copy and complete with <, >, or =.

25. $m\angle BAC$ __?__ $m\angle DAC$ >



26. LM __?__ KN =



27. Arrange statements A–D in correct order to write an indirect proof of the statement: *If two lines intersect, then their intersection is exactly one point.* C, B, A, D

GIVEN ▶ Intersecting lines m and n

PROVE ▶ The intersection of lines m and n is exactly one point.

A. But this contradicts Postulate 5, which states that through any two points there is exactly one line.

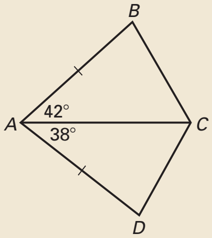
B. Then there are two lines (m and n) through points P and Q.

C. Assume that there are two points, P and Q, where m and n intersect.

D. It is false that m and n can intersect in two points, so they must intersect in exactly one point.

Extra Example 6

How does the length of $\overline{CD}$ compare to the length of $\overline{CB}$? $CD < CB$



Additional Resources

Assessment Book

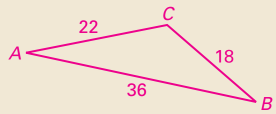
- Chapter Test, Levels A, B, C
- Standardized Chapter Test
- SAT/ACT Chapter Test
- Alternative Assessment

ExamView™ Assessment Suite

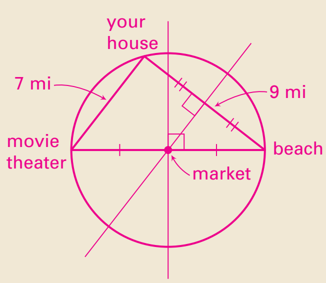
Chapter Test

Easily-readable reduced copies of Chapter Test B, the Standardized Chapter Test, and the Alternative Assessment from the Assessment Book can be found at the beginning of this chapter.

13.



18.



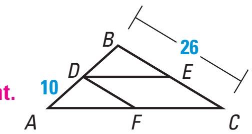
5 CHAPTER TEST

4. 2; $\triangle SWV \cong \triangle UWV$ so $SV = UV$.

5. 3; since $\overline{QS}$ bisects $\angle PSR$ the Angle Bisector Theorem guarantees $PQ = RQ$.

6. 7; since J is interior to $\angle HGK$ and equidistant from each side of the angle, the Converse of the Angle Bisector Theorem guarantees $m\angle HGJ = m\angle KGJ$.

Two midsegments of $\triangle ABC$ are $\overline{DE}$ and $\overline{DF}$.



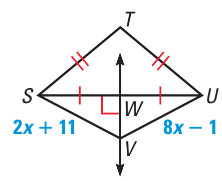
1. Find DB. **10**

2. Find DF. **13**

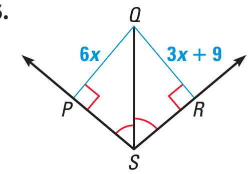
3. What can you conclude about $\overline{EF}$? **$\overline{EF}$ is a midsegment.**

Find the value of x. *Explain* your reasoning.

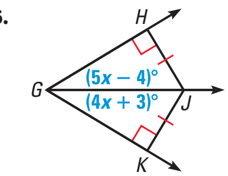
4.



5.

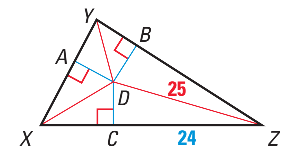


6.

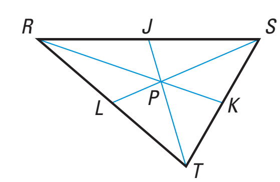


7. In Exercise 4, is point T on the perpendicular bisector of $\overline{SU}$? *Explain.* **Yes; $\triangle STU$ is isosceles.**

8. In the diagram at the right, the angle bisectors of $\triangle XYZ$ meet at point D. Find DB. **7**



In the diagram at the right, P is the centroid of $\triangle RST$.



9. If $LS = 36$, find PL and PS. **12, 24**

10. If $TP = 20$, find TJ and PJ. **30, 10**

11. If $JR = 25$, find JS and RS. **25, 50**

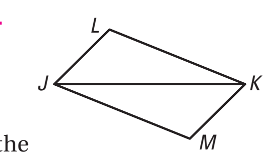
12. Is it possible to construct a triangle with side lengths 9, 12, and 22? If not, *explain* why not. **No; the sum of the lengths of any two sides of a triangle must be greater than the length of the third side.**

13. In $\triangle ABC$, $AB = 36$, $BC = 18$, and $AC = 22$. Sketch and label the triangle. List the angles in order from smallest to largest. **$\angle A$, $\angle B$, $\angle C$; see margin for art.**

14. $\overline{MJ}$; the Hinge Theorem guarantees the longer side is opposite the larger angle.

15. $\angle LJK$; the Converse of the Hinge Theorem guarantees the larger angle is opposite the longer side.

In the diagram for Exercises 14 and 15, $JL = MK$. **See margin.**

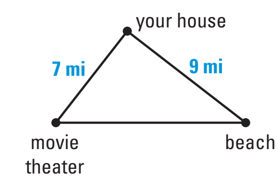


14. If $m\angle JKM > m\angle LJK$, which is longer, $\overline{LK}$ or $\overline{MJ}$? *Explain.*

15. If $MJ < LK$, which is larger, $\angle LJK$ or $\angle JKM$? *Explain.*

16. Write a temporary assumption you could make to prove the conclusion indirectly: *If $RS + ST \neq 12$ and $ST = 5$, then $RS \neq 7$.* **Assume that $RS = 7$.**

Use the diagram in Exercises 17 and 18.



17. *Describe* the range of possible distances from the beach to the movie theater. **2 mi < d < 16 mi**

18. A market is the same distance from your house, the movie theater, and the beach. Copy the diagram and locate the market. **See margin.**

5 xy ALGEBRA REVIEW

Animated Algebra my.hrw.com

USE RATIOS AND PERCENT OF CHANGE

EXAMPLE 1 *Write a ratio in simplest form*

A team won 18 of its 30 games and lost the rest. Find its win-loss ratio.

The ratio of a to b, $b \neq 0$, can be written as a to b, $a:b$, and $\frac{a}{b}$.

$$\frac{\text{wins}}{\text{losses}} = \frac{18}{30-18}$$ To find losses, subtract wins from total.

$$= \frac{18}{12} = \frac{3}{2}$$ Simplify.

▶ The team's win-loss ratio is 3 : 2.

EXAMPLE 2 *Find and interpret a percent of change*

A \$50 sweater went on sale for \$28. What is the percent of change in price? The new price is what percent of the old price?

$$\text{Percent of change} = \frac{\text{Amount of increase or decrease}}{\text{Original amount}} = \frac{50-28}{50} = \frac{22}{50} = 0.44$$

▶ The price went down, so the change is a decrease. The percent of decrease is 44%. So, the new price is 100%−44% = 56% of the original price.

EXERCISES

EXAMPLE 1 for Exs. 1–3

1. A team won 12 games and lost 4 games. Write each ratio in simplest form.
   a. wins to losses $\frac{3}{1}$
   b. losses out of total games $\frac{1}{4}$
2. A scale drawing that is 2.5 feet long by 1 foot high was used to plan a mural that is 15 feet long by 6 feet high. Write each ratio in simplest form.
   a. length to height of mural $\frac{5}{2}$
   b. length of scale drawing to length of mural $\frac{1}{6}$
3. There are 8 males out of 18 members in the school choir. Write the ratio of females to males in simplest form. $\frac{5}{4}$

EXAMPLE 2 for Exs. 4–13

Find the percent of change.

4. From 75 campsites to 120 campsites **60% increase**
5. From 150 pounds to 136.5 pounds **9% decrease**
6. From \$480 to \$408 **15% decrease**
7. From 16 employees to 18 employees **12.5% increase**
8. From 24 houses to 60 houses **150% increase**
9. From 4000 ft^2 to 3990 ft^2 **0.25% decrease**

Write the percent comparing the new amount to the original amount. Then find the new amount.

10. 75 feet increased by 4% **104%; 78 ft**
11. 45 hours decreased by 16% **84%; 37.8 h**
12. \$16,500 decreased by 85% **15%; \$2475**
13. 80 people increased by 7.5% **107.5%; 86 people**

Extra Example 1

A softball player got 180 hits out of 500 times she batted. Find the ratio of the number of times she got a hit to the number of times she did not get a hit. **9:16**

Extra Example 2

A \$240 MP3 music player was on sale for \$180. What is the percent of change in price? The new price is what percent of the old price? **25%; 75%**

Using Rubrics

The rubric given on the pupil page is a sample of a three-level rubric. Other rubrics may contain four, five, or six levels. For more information on rubrics, see the *Differentiated Instruction Resources*.

Test-Taking Strategy

Drawing a graph when given coordinates is a useful strategy as it allows students to see what the figure appears to be (in this case, an equilateral triangle). Given the graph, students' calculations should indicate that the triangle is equilateral (or something close to it). If this is not the case, students should realize that either the graph or the calculations are incorrect.

Avoiding Common Errors

Students may simplify $\sqrt{(k-0)^2 + (k\sqrt{3}-0)^2}$ as $\sqrt{k^2} + \sqrt{(k\sqrt{3})^2} = k + k\sqrt{3}$. Remind them that $\sqrt{a^2+b^2} \neq \sqrt{a^2} + \sqrt{b^2}$ for nonzero values of a and b.

Study Strategy

Tell students to start by using sample values of k in points O, M, and N to practice using the distance formula. Then have them substitute in the variable k and calculate the distance formula with k.

Avoiding Common Errors

For the practice problem, students may assume that since A is closer to $\overline{NQ}$, then position A is automatically the correct answer. Remind them that the problem depends on properties of an angle bisector, so those properties must be considered.

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,
  or
- solution is without error but incomplete

No Credit
- no solution is given,
  or
- solution makes no sense

SHORT RESPONSE QUESTIONS

PROBLEM

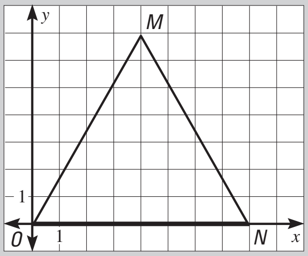
The coordinates of the vertices of a triangle are $O(0, 0)$, $M(k, k\sqrt{3})$, and $N(2k, 0)$. Classify $\triangle OMN$ by its side lengths. *Justify* your answer.

Below are sample solutions to the problem. Read each solution and the comments on the left to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

A sample triangle is graphed and an explanation is given.

Begin by graphing $\triangle OMN$ for a given value of k. I chose a value of k that makes $\triangle OMN$ easy to graph. In the diagram, $k = 4$, so the coordinates are $O(0, 0)$, $M(4, 4\sqrt{3})$, and $N(8, 0)$.



From the graph, it appears that $\triangle OMN$ is equilateral.

The Distance Formula is applied correctly.

To verify that $\triangle OMN$ is equilateral, use the Distance Formula. Show that $OM = MN = ON$ for all values of k.

$$OM = \sqrt{(k-0)^2 + (k\sqrt{3}-0)^2} = \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2|k|$$

$$MN = \sqrt{(2k-k)^2 + (0-k\sqrt{3})^2} = \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2|k|$$

$$ON = \sqrt{(2k-0)^2 + (0-0)^2} = \sqrt{4k^2} = 2|k|$$

The answer is correct.

Because all of its side lengths are equal, $\triangle OMN$ is an equilateral triangle.

SAMPLE 2: Partial credit solution

Use the Distance Formula to find the side lengths.

A calculation error is made in finding OM and MN. The value of $(k\sqrt{3})^2$ is $k^2 \cdot (\sqrt{3})^2$, or $3k^2$, not $9k^2$.

$$OM = \sqrt{(k-0)^2 + (k\sqrt{3}-0)^2} = \sqrt{k^2 + 9k^2} = \sqrt{10k^2} = k\sqrt{10}$$

$$MN = \sqrt{(2k-k)^2 + (0-k\sqrt{3})^2} = \sqrt{k^2 + 9k^2} = \sqrt{10k^2} = k\sqrt{10}$$

$$ON = \sqrt{(2k-0)^2 + (0-0)^2} = \sqrt{4k^2} = 2k$$

The answer is incorrect.

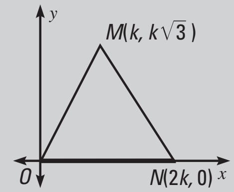
Two of the side lengths are equal, so $\triangle OMN$ is an isosceles triangle.

SAMPLE 3: Partial credit solution

The answer is correct, but the explanation does not justify the answer.

Graph $\triangle OMN$ and compare the side lengths.

From $O(0, 0)$, move right k units and up $k\sqrt{3}$ units to $M(k, k\sqrt{3})$. Draw $\overline{OM}$. To draw $\overline{MN}$, move k units right and $k\sqrt{3}$ units down from M to $N(2k, 0)$. Then draw $\overline{ON}$, which is $2k$ units long. All side lengths appear to be equal, so $\triangle OMN$ is equilateral.



SAMPLE 4: No credit solution

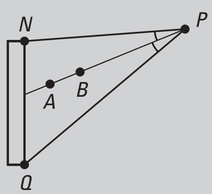
The reasoning and the answer are incorrect.

You are not given enough information to classify $\triangle OMN$ because you need to know the value of k.

PRACTICE Apply the Scoring Rubric

Use the rubric to score the solution to the problem below as *full credit*, *partial credit*, or *no credit*. *Explain* your reasoning.

PROBLEM You are a goalie guarding the goal $\overline{NQ}$. To make a goal, Player P must send the ball across $\overline{NQ}$. Is the distance you may need to move to block the shot greater if you stand at Position A or at Position B? *Explain*.

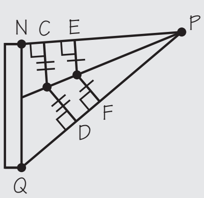


1. At either position, you are on the angle bisector of ∠NPQ. So, in both cases you are equidistant from the angle's sides. Therefore, the distance you need to move to block the shot from the two positions is the same.

2. Both positions lie on the angle bisector of ∠NPQ. So, each is equidistant from $\overline{PN}$ and $\overline{PQ}$.

   The sides of an angle are farther from the angle bisector as you move away from the vertex. So, A is farther from $\overline{PN}$ and from $\overline{PQ}$ than B is.

   The distance may be greater if you stand at Position A than if you stand at Position B.



3. Because Position B is farther from the goal, you may need to move a greater distance to block the shot if you stand at Position B.

Answers

1. Partial credit; the initial set-up of the problem is correct but no diagram is provided and the conclusion reached is incorrect.

2. Full credit; the initial set-up of the problem is correct, a correct diagram is provided, and the answer is correct.

3. No credit; the reasoning and answer are incorrect.

Answers

1. Isosceles; yes; the slope of $\overline{PQ}$ is -1 and the slope of $\overline{OP}$ is 1.

2. 20 ft; the sides of the small triangle are the midsegments of the bigger triangle. You know that the perimeter of the bigger one is twice the perimeter of the smaller one by using the Midsegment Theorem.

3. Less than; if the support were the midsegment, it would measure 8 inches; since it is above the midsegment, it has to be smaller.

4. The sum of any two sides of a triangle must be greater than the third. In this example, $\frac{3}{4}$ in. $+ \frac{5}{8}$ in. $< 1\frac{1}{2}$ in.

5. $(-1, -4)$; the median from C is a segment on the vertical line $x = -1$. Since $(-1, 5)$ is the midpoint of $\overline{AB}$, the distance to the centroid is 3 units making point C 6 units from the centroid.

6. B; Concurrency of Perpendicular Bisectors Theorem

7. A; the Hinge Theorem guarantees $\overline{AP}$ is shorter than $\overline{BP}$.

8. It is $\frac{1}{4}$ the area of the original triangle. *Sample answer:* Consider $\triangle ABC$ with vertices $A(2a, 0)$, $B(-2a, 0)$, and $C(0, 2a)$ whose area is $4a^2$ and $\triangle DEF$ (the triangle formed by the midsegments of $\triangle ABC$) with vertices $D(a, a)$, $E(0, 0)$, and $F(-a, a)$ whose area is a^2.

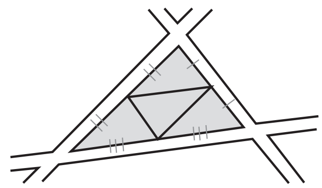
9. By Theorem 5.11, $\ell > 6$ in. By the Triangle Inequality Theorem, $\ell < 9$ in.

5 ★ Standardized TEST PRACTICE

SHORT RESPONSE

1. The coordinates of $\triangle OPQ$ are $O(0, 0)$, $P(a, a)$, and $Q(2a, 0)$. Classify $\triangle OPQ$ by its side lengths. Is $\triangle OPQ$ a right triangle? *Justify* your answer.

2. The local gardening club is planting flowers on a traffic triangle. They divide the triangle into four sections, as shown. The perimeter of the middle triangle is 10 feet. What is the perimeter of the traffic triangle? *Explain* your reasoning.

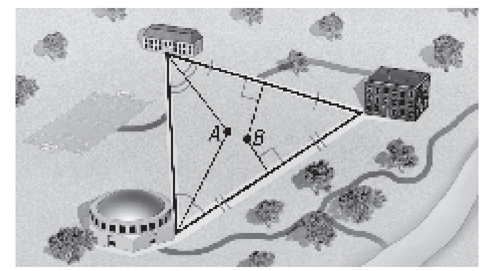


3. A wooden stepladder with a metal support is shown. The legs of the stepladder form a triangle. The support is parallel to the floor, and positioned about five inches above where the midsegment of the triangle would be. Is the length of the support from one side of the triangle to the other side of the triangle *greater than*, *less than*, or *equal to* 8 inches? *Explain* your reasoning.

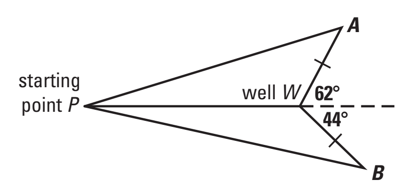
4. You are given instructions for making a triangular earring from silver wire. According to the instructions, you must first bend a wire into a triangle with side lengths of $\frac{3}{4}$ inch, $\frac{5}{8}$ inch, and $1\frac{1}{2}$ inches. *Explain* what is wrong with the first part of the instructions.

5. The centroid of $\triangle ABC$ is located at $P(-1, 2)$. The coordinates of A and B are $A(0, 6)$ and $B(-2, 4)$. What are the coordinates of vertex C? *Explain* your reasoning.

6. A college club wants to set up a booth to attract more members. They want to put the booth at a spot that is equidistant from three important buildings on campus. Without measuring, decide which spot, A or B, is the correct location for the booth. *Explain* your reasoning.



7. Contestants on a television game show must run to a well (point W), fill a bucket with water, empty it at either point A or B, and then run back to the starting point (point P). To run the shortest distance possible, which point should contestants choose, A or B? *Explain* your reasoning.



8. How is the area of the triangle formed by the midsegments of a triangle related to the area of the original triangle? Use an example to *justify* your answer.

9. You are bending an 18 inch wire to form an isosceles triangle. *Describe* the possible lengths of the base if the vertex angle is larger than 60°. *Explain* your reasoning.

MULTIPLE CHOICE

10. If $\triangle ABC$ is obtuse, which statement is always true about its circumcenter P?

Ⓐ P is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.

Ⓑ P is inside $\triangle ABC$.

Ⓒ P is on $\triangle ABC$.

Ⓓ P is outside $\triangle ABC$.

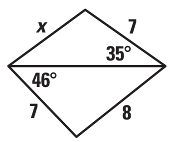
11. Which conclusion about the value of x can be made from the diagram?

Ⓐ $x < 8$

Ⓑ $x = 8$

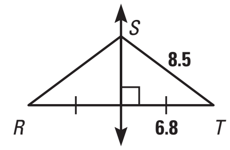
Ⓒ $x > 8$

Ⓓ No conclusion can be made.

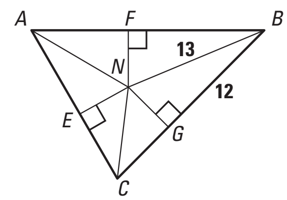


GRIDDED ANSWER

12. Find the perimeter of $\triangle RST$.



13. In the diagram, N is the incenter of $\triangle ABC$. Find NF.

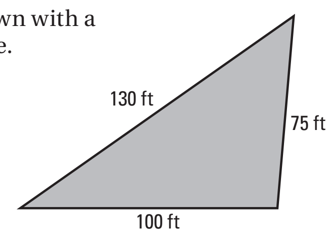


EXTENDED RESPONSE

14. A new sport is to be played on the triangular playing field shown with a basket located at a point that is equidistant from each side line.

a. Copy the diagram and show how to find the location of the basket. *Describe* your method.

b. What theorem can you use to verify that the location you chose in part (a) is correct? *Explain*.



15. A segment has endpoints $A(8, -1)$ and $B(6, 3)$.

a. Graph $\overline{AB}$. Then find the midpoint C of $\overline{AB}$ and the slope of $\overline{AB}$.

b. Use what you know about slopes of perpendicular lines to find the slope of the perpendicular bisector of $\overline{AB}$. Then sketch the perpendicular bisector of $\overline{AB}$ and write an equation of the line. *Explain* your steps.

c. Find a point D that is a solution to the equation you wrote in part (b). Find AD and BD. What do you notice? What theorem does this illustrate?

16. The coordinates of $\triangle JKL$ are $J(-2, 2)$, $K(4, 8)$, and $L(10, -4)$.

a. Find the coordinates of the centroid M. Show your steps.

b. Find the mean of the x-coordinates of the three vertices and the mean of the y-coordinates of the three vertices. *Compare* these results with the coordinates of the centroid. What do you notice?

c. Is the relationship in part (b) true for $\triangle JKP$ with $P(1, -1)$? *Explain*.

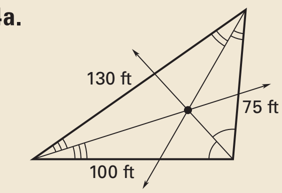
10. D

11. A

12. 30.6

13. 5

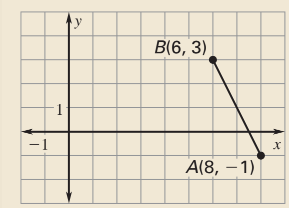
14a.



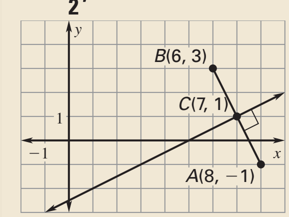
The angle bisectors will intersect in the point that is equidistant from each side.

14b. Concurrency of Angle Bisectors Theorem; it states that the angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

15a. ***C*(7, 1), 22;**



15b. $\frac{1}{2}$**;**



$y = \frac{1}{2}x - \frac{5}{2}$**; using the point (7, 1) and slope** $\frac{1}{2}$ **you get** $y - 1 = \frac{1}{2}(x - 7)$.

15c. ***Sample answer:*** **(5, 0);** $\sqrt{10}$, $\sqrt{10}$; $AD = BD$**; Perpendicular Bisector Theorem**

16a. (4, 2). ***Sample answer:*** **The equation of the line passing through midpoint (1, 5) of** $\overline{JK}$ **and** L **with slope −1 is** $y = -x + 6$**. The equation of the line passing through the midpoint (4, −1) of** $\overline{JL}$ **and** K **with undefined slope is** $x = 4$**. The intersection is the centroid.**

16b. 4, 2; they are the same as the *x*- and *y*-coordinates of the centroid.

16c. Yes; the centroid of $\triangle JKP$ **is (1, 3). The mean of the *x*-coordinates is 1 and the mean of the *y*-coordinates is 3.**